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# Three essays on renewable backstop technology

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**Three essays on renewable backstop technology**

by

**Jittinan Aukayanagul**

A dissertation submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of

**DOCTOR OF PHILOSOPHY**

Major: Economics

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## ABSTRACT

This dissertation comprises three essays on backstop technology as a key to weak sustainability of commodity resources. Through the use of a basic model of renewable ground water, the first essay separately looks at water scarcity problems posed by growing water demand and stochastic rainfall. The role of artificial ground water recharge in augmenting the ground water supply is examined. The second essay looks at a long-run relationship between the prices of two substitutable resources, ethanol and oil, and tests the hypothesis that the derived demand for fuel ethanol in the US is perfectly elastic. The Johansen and Jesulius multivariate cointegration methodology finds no cointegration between the ethanol price and the gasoline price while the Gregory and Hansen residual-based tests for cointegration in models with regime shifts indicate that the long-run relationship between the ethanol price and the gasoline price exists with a possible structural break. The third essay looks at water recycling in ethanol production as a means to reduce some of the ethanol pressure on the (ground) water resources. Although modern ethanol plants possess sophisticated water treatment techniques for water recycling, water recycling is done only when it is cheaper than obtaining water from the outside source. Since water recycling can lower the cost of production, it may adversely induce production expansion and lead to more outside water being used by the plants. The conditions under which this possibility occurs are examined.

## CHAPTER 1. GENERAL INTRODUCTION

### 1 Introduction

The first lesson in economics is scarcity. Human wants are unlimited, whereas the means to fulfill them are not. Just as individuals do not have enough income to meet all of their desires, an entire economy also faces resource scarcity. Given unprecedented growth in natural resource consumption in the US over the past two centuries and finite supplies of natural resources, a question arises whether the future resource supplies will be sufficient to sustain economic growth (Krautkraemer, 2005). In order to answer this question, a distinction must be made between commodity resources and amenity resources. Commodity resources are used to produce material goods and services. Since they have a lot of close substitutes and can be enhanced by technology, they do not need to be physically maintained to be sustainable as long as the rents derived from their use are reinvested. Amenity resources, on the other hand, provide recreational benefits and environmental services to people. Since their supplies are fixed and they are irreplaceable, they need to be physically maintained to be sustainable. This dissertation looks at the role of backstop technology in sustainability of commodity resources. As resources become scarcer, their price increases and signals a switch to a relatively more expensive renewable substitute, which is sometimes called a backstop technology.

Specifically, the first essay separately looks at water scarcity problems posed by growing water demand and stochastic rainfall. The role of artificial ground water recharge in augmenting the ground water supply is examined. Though ground water is replenishable, it can be depleted if withdrawals exceed recharge for a long period of time. When that occurs, other sources of water may be needed. A basic model of renewable ground water is extended to see how different sources of water could be used to maximize the net benefits of water consumption over time.

The second essay looks at a long-run relationship between the prices of two substitutable resources, ethanol and oil, and tests the hypothesis that the derived demand for fuel ethanol in the US is perfectly elastic. The Johansen and Juselius multivariate cointegration methodology is used to examine whether the ethanol price and the gasoline



price are cointegrated. The Gregory and Hansen (1996) residual-based tests for cointegration in models with regime shift are utilized to see if a long-run equilibrium ethanol price equation exists with a structural break.

The third essay examines the possibility of water recycling in a corn-based fuel ethanol plant leading to more outside water being used by the plant due to production expansion caused by changes in the cost structure of the plant in the presence of water recycling. A static model of a profit maximizing fuel ethanol plant is used in making a comparison between the amount of outside water withdrawn by the plant when there is no water recycling and the amount of outside water withdrawn by the plant when there is water recycling. Alternative recycling incentives and their impacts on outside water use are also considered.

## **2 Research Motivation**

The motivation for the dissertation comes from an article in a newspaper about the use of Ada Hayden lake as a potable water supply back up in Ames, Iowa (Zientara, 2006, p. F1). The city of Ames relies on a ground water system for its potable water. So it would be interesting to see when and how water should be pumped from the lake to a ground water recharge area to artificially recharge the aquifer so that the net benefits of water consumption are maximized over time. Besides potable water use, there are other uses of water in Ames. One of which is water use by ethanol plants. A considerable amount of water required for ethanol production can put a strain on the local (ground) water sources. So it would be interesting to see how water recycling could help reduce the amount of outside water withdrawn by the plants. As ethanol is used in the US as an oxygenate, an octane enhancer, and a gasoline volume extender, the demand for fuel ethanol can be considered as being derived from both government regulations, which mandate oxygenate use, and the gasoline market. So it would be interesting to look at a relationship between the ethanol price and the gasoline price.

## **3 Dissertation Organization**

This dissertation is organized as follows. The next chapter contains the first essay, “water supply system: potable water and artificial ground water recharge.” A basic model of

renewable ground water with artificially recharged ground water and water from other cities as backstop technologies is provided. Chapter 3 contains the second essay, “US fuel ethanol demand,” and provides a vector error correction model (VECM) used in finding a cointegrating relationship between the ethanol price and the gasoline price. Chapter 4 contains the third essay, “water recycling in fuel ethanol plant,” and provides a static model of fuel ethanol production used in examining water recycling in fuel ethanol plant. Chapter 5 concludes.

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## CHAPTER 2. WATER SUPPLY SYSTEM: POTABLE WATER AND ARTIFICIAL GROUND WATER RECHARGE

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Jittinan Aukayanagul

### Abstract

The model of renewable ground with backstop is extended to study the role of artificial ground water recharge in augmenting ground water supply. The impacts of growing water demand and stochastic rainfall, i.e. the key factors for water scarcity, on intertemporal potable water use are separately examined.

### 1 Introduction

Turn on a water faucet and out comes the water one needs. The water supply may seem endless. But a cheap supply is not. A question arises as to how different sources of water might be used so that the discounted net benefits of water consumption are maximized. The problem of intertemporal water management has attracted much research over time. Much of the literature focuses on the agricultural aspect of the water use. For instance, Gisser and Mercado (1973) looks at intertemporal ground water management problem in a semiarid agricultural region, integrating a demand function for irrigation water with a hydrologic model of an aquifer. Surface water is used to artificially recharge the aquifer. Deterministic natural recharge of the aquifer is assumed. Tsur (1991) studies management of an irrigation and drainage system where water comes from both ground and surface water sources. Ground water and surface water are assumed perfect substitutes in the water response function. Direct artificial recharge of ground water is not allowed. However, a fraction of ground water and surface water applied for irrigation is assumed to permeate into the aquifer. The amount of rainfall is treated as a constant and included as part of surface water applied for irrigation. In some cases, attention is paid to the non-agricultural aspect of the water use. Krulce, Roumasset, and Wilson (1997) looks at the intertemporal potable water management problem in a coastal area of Hawaii, where water comes primarily from

an aquifer and where costly desalinated oceanic water is the infinite backstop. Fixed inflow to the aquifer from rainfall is assumed. The issue of growing water demand is also addressed.

As can be seen, the natural addition to the ground water stock or the rainfall is often treated as constant or known. Despite the abundant literature on uncertainty in the theory of renewable resources, e.g. Pindyck (1980); Pindyck (1984); Hertzler (1991); Slade and Thille (1997); and Costello and Polasky (2006), the stochastic aspect of the intertemporal water management problem has not been examined by very many studies. Among exceptions is Burt (1964) where a general stochastic renewable resource allocation model is applied to ground water storage control. The effects of changes in the expected natural addition to the ground water stock on the optimal ground water consumption are analyzed. Another exception is Palma (2004) which extends the deterministic models of conjunctive ground and surface water management to ones in which there exist quality difference between the two water sources and uncertainty in surface water availability. Surface water is treated as exogenous so that by choosing total water use the amount of ground water extracted is established. A portion of the used water is assumed to infiltrate the aquifer. However, no direct artificial ground water recharge is allowed.

This paper looks at the intertemporal water management problem in a city where ground water is the primary source of potable water. Surface water is the city's secondary source. However, it is used only to artificially recharge the aquifer. This is based on the assumption that surface water is not as clean as ground water and can easily become contaminated. Since bacteria and other potential disease-causing agents are often absorbed and filtered out of ground water, the final treatment of artificially recharged ground water, if necessary, becomes much easier and cheaper than that of surface water (Balke & Zhu, 2008). Among areas where artificial ground water recharge is known to exist are localities in Arizona; Los Angeles, Orange County, and Fresno, California; Alachua County, Florida; Ames, Iowa; Long Island and Nassau County, New York, and the High Plains States.<sup>1</sup>

<sup>1</sup> Denver Basin, Colorado; Equus Beds, Kansas; Wood River and York, Nebraska; Turner-Hogeland, Montana; Blaine Gypsum, Oklahoma; Hueco Bolson, Texas; and Huron, South Dakota

Surface water spreading is one of the simplest and the most widely used artificial ground water recharge methods in these areas.<sup>2</sup> The city's third source of water is to purchase clean-and-ready-to-use water from other cities where rainfall is spatially uncorrelated.<sup>3</sup> The paper extends Krulce et al.'s (1997) model of renewable ground water under growing water demand and constant rainfall to study the role of artificial ground water recharge in augmenting the ground water supply. The impact of stochastic rainfall (i.e. another key factor for water scarcity besides growing water demand) on intertemporal water use is also examined. However, since solving model which accounts for both growing water demand and stochastic rainfall can prove challenging, constant water demand is assumed when examining the impact of stochastic rainfall on intertemporal water use.

The paper is organized in the following way. A basic model of renewable ground water with artificially recharged ground water and water from other cities as backstop technologies is presented in the next section. The third section examines the impact of growing water demand on intertemporal water use. To focus on water scarcity problem posed by growing water demand, constant rainfall is assumed. A numerical example for Ames, Iowa is provided to show specifically the optimal drawdown of ground water in its transition to steady state and the artificial ground water recharge path. The fourth section deals with stochastic rainfall and its implication on intertemporal water use. To focus on water scarcity problem posed by stochastic rainfall, the potential time dependence of the water demand is ignored. In other words, the water benefit function is assumed not to change over time. For illustrative purpose, a numerical example for Ames, Iowa is also provided. The fifth section concludes.

## 2 Model

Notations used in the model are as follows. Let  $G(t)$  be the stock of ground water at time  $t$  and  $G_0$  be the initial stock of ground water. Extracting ground water at lower stock

<sup>2</sup> Surface water spreading is possibly done via ponds, check dams, pits, furrows, or ditches and involves releasing water over ground surface to increase the quantity of water infiltrating into the ground and percolating down to a shallow, unconfined aquifer.

<sup>3</sup> Another option could be to put in a water treatment plant to take advantage of quarries and other surface water sources in the area. For cities close to the ocean, water desalination may be used (Krulce et al., 1997).

levels requires deeper drilling of the wells, as well as water to be lifted greater distances. Therefore, the marginal ground water extraction cost  $C^g(G(t))$  is a positive, decreasing, convex function of the ground water stock. In other words, it is assumed that  $C^g(G(t)) > 0$ ,  $C^g_G(G(t)) < 0$ ,  $C^g_{GG}(G(t)) > 0$ , and  $\lim_{G(t) \rightarrow 0} C^g(G(t)) = \infty$ . As the aquifer gets close to exhaustion ( $G(t) = 0$ ), the extraction cost rises rapidly.

In each period, an amount  $g(t)$  of ground water is drawn from the aquifer. At the same time, a fraction  $\tau < 1$  of the rainfall  $R(t)$  permeates into the aquifer. Moreover, an amount  $a(t)$  can be pumped from the lakes or any other surface water sources at a constant marginal cost  $C^a$  to spreading basins<sup>4</sup> to artificially recharge the aquifer. However, only a fraction  $\omega < 1$  of that amount percolates down to the aquifer. This is because the amount of water entering the aquifer by surface water spreading depends on the infiltration rate of soil,<sup>5</sup> the percolation rate of soil,<sup>6</sup> and the capacity for horizontal water movement (O'Hare, Fairchild, Hajali, & Canter, 1986, pp. 1-28). Since recharge structures such as ponds, check dams, pits, furrows, or ditches are needed for surface water spreading, there may be a limit to the amount of water maintained over the spreading basins at each time. In addition, there may be limits to the capacities of the pumping equipments and, as the lakes may be used for fishing and other water-based activities, a limit to the amount of water pumped from the lakes to the spreading basins. Therefore, it is assumed that there is an upper limit  $\bar{a}$  on the artificial recharge from the lakes, i.e.  $a(t) < \bar{a}$ . As a result, the ground water stock evolves over time as  $dG(t) = \{\tau R(t) + \omega a(t) - g(t)\}dt$ . Due to today's high water demand, the aquifer is assumed to never fill up.

Besides the ground water, clean-and-ready-to-use water may be brought in from other cities at a high marginal cost  $C^b$ . Let  $b(t)$  be water drawn from this backstop source. Because the ground water and the water obtained from other cities are treated as perfect

<sup>4</sup> These are areas with exceedingly permeable soil used to artificially recharge ground water.

<sup>5</sup> This is the rate at which water on the ground surface enters the soil.

<sup>6</sup> This is the rate at which water is able to move downward through the soil. It depends on vertical hydraulic conductivity (i.e. a measure of soil's ability to transmit water when submitted to hydraulic gradient).

substitutes in terms of quality,<sup>7</sup> the total benefit associated with water use is  $B(g(t) + b(t), t)$ . This benefit function is allowed to change over time and is positive, increasing, and concave in the amount of water used. In other words, it is assumed that  $B(g(t) + b(t), t) > 0$ ,  $B_1(g(t) + b(t), t) > 0$ , and  $B_{11}(g(t) + b(t), t) < 0$ . Also because the city needs water, one has that  $\lim_{g(t)+b(t) \rightarrow 0} B_1(g(t) + b(t), t) = \infty$ .

Given a real social discount rate  $r > 0$ , the social planner chooses the extraction rate of ground water, the artificial ground water recharge rate, and the use of desalinated oceanic water to maximize the expected present value of net social surplus associated with water use, assuming all the constraints are met. The social planner's optimization problem is characterized as follows:

$$\max_{g(t), a(t), b(t), G(t)} E_0 \left\{ \int_0^{\infty} e^{-rt} [B(g(t) + b(t), t) - C^g(G(t))g(t) - C^a a(t) - C^b b(t)] dt \right\} \quad (2.1)$$

$$\text{subject to } dG(t) = \{\tau R(t) + \omega a(t) - g(t)\} dt$$

$$a(t) \leq \bar{a}$$

$$g(t), a(t), b(t) \geq 0$$

$$G(0) = G_0$$

### 3 Growing Water Demand

This section examines the impact of growing water demand on the intertemporal water use. The water demand may grow over time because of increasing population, rising income, and growing general economic activities in the area. To focus on water scarcity problem posed by growing water demand, constant rainfall is assumed.

#### 3.1 Optimal Rules

With constant rainfall  $R$ , the current value Hamiltonian for system (2.1) is:

$$H(t) = \left[ B(g(t) + b(t), t) - C^g(G(t))g(t) - C^a a(t) - C^b b(t) \right] + \delta(t) \{\bar{a} - a(t)\} + \lambda(t) \{\tau R + \omega a(t) - g(t)\} \quad (3.1.1)$$

where  $\lambda(t) \geq 0$ . The necessary conditions for an optimal solution are:

$$\frac{\partial H(t)}{\partial g(t)} = B_1(g(t) + b(t), t) - C^g(G(t)) - \lambda(t) \leq 0, \quad g(t) \geq 0, \quad \frac{\partial H(t)}{\partial g(t)} g(t) = 0 \quad (3.1.2)$$

<sup>7</sup> The quality difference of the two may come in terms of different extraction costs.

$$\frac{\partial H(t)}{\partial a(t)} = -C^a - \delta(t) + \omega\lambda(t) \leq 0, \quad a(t) \geq 0, \quad \frac{\partial H(t)}{\partial a(t)} a(t) = 0 \quad (3.1.3)$$

$$\frac{\partial H(t)}{\partial \delta(t)} = \bar{a} - a(t) \geq 0, \quad \delta(t) \geq 0, \quad \frac{\partial H(t)}{\partial \delta(t)} \delta(t) = 0 \quad (3.1.4)$$

$$\frac{\partial H(t)}{\partial b(t)} = B_1(g(t) + b(t), t) - C^b \leq 0, \quad b(t) \geq 0, \quad \frac{\partial H(t)}{\partial b(t)} b(t) = 0 \quad (3.1.5)$$

$$\dot{\lambda}(t) = r\lambda(t) - \frac{\partial H(t)}{\partial G(t)} = r\lambda(t) + C_G^g(G(t))g(t) \quad (3.1.6)$$

$$\dot{G}(t) = \tau R + \omega a(t) - g(t) \quad (3.1.7)$$

$$\lim_{t \rightarrow \infty} e^{-rt} \lambda(t) = 0 \quad (3.1.8)$$

$$G(0) = G_0 \quad (3.1.9)$$

Before proceeding, it is useful to think of the marginal benefit  $B_1(g(t) + b(t), t)$  as an inverse demand. Let's define  $p(t) \equiv B_1(g(t) + b(t), t)$ , where  $p(t)$  is the optimal price at time  $t$ .

Again, the water demand is assumed to grow over time. Since one cannot live without water,

i.e.  $\lim_{g(t)+b(t) \rightarrow 0} B_1(g(t) + b(t), t) = \infty$ , and if the cost of obtaining water from other cities  $C^b$  is

sufficiently high, ground water is always extracted ( $g(t) > 0$ ). From (3.1.2), one has:

$$p(t) = C^g(G(t)) + \lambda(t) \quad (3.1.10)$$

The marginal benefit of extracting ground water is equal to the marginal cost which breaks down into marginal extraction cost and marginal user cost. (3.1.10) provides the optimal rule for the ground water extraction. Note, however, that the social optimal rule may not be achieved if the ground water use is left to the market (i.e. the market valuation of  $\lambda(t)$  equals zero because of the common property nature of the ground water).

(3.1.6) must hold for all cases, i.e. whether water is pumped from the lakes and/or obtained from other cities. Rearranging (3.1.6) yields the following:

$$\dot{\lambda}(t) - C_G^g(G(t))g(t) = r\lambda(t) \quad (3.1.11)$$

This is simply an arbitrage condition, stating that the change in the marginal user cost from not consuming  $g(t)$  plus the reduction in the future extraction cost from the increase in the stock of ground water by  $g(t)$  must equal the interest amount on the benefit that would have been gained should  $g(t)$  be consumed. In other words, the benefit of extracting ground water must equal the cost at the margin.



From (3.1.3) and (3.1.4), the following can be obtained:

$$\text{if } \lambda(t) < \frac{c^a}{\omega}, \text{ then } a(t) = 0. \quad (3.1.12)$$

$$\text{if } \lambda(t) = \frac{c^a}{\omega}, \text{ then } 0 \leq a(t) \leq \bar{a}. \quad (3.1.13)$$

$$\text{if } \lambda(t) > \frac{c^a}{\omega}, \text{ then } a(t) = \bar{a}. \quad (3.1.14)$$

(3.1.12), (3.1.13), and (3.1.14) provide the optimal rule for the artificial recharge of the aquifer. Water is pumped from the lakes if the net marginal benefit of having a unit more of water underground is equal to or greater than the unit cost of water that actually goes down to the aquifer,  $\lambda(t) = p(t) - C^g(G(t)) \geq \frac{c^a}{\omega}$ .

Rewriting (3.1.5) yields the following:

$$p(t) \leq C^b, \quad b(t) \geq 0, \quad \{p(t) - C^b\}b(t) = 0 \quad (3.1.15)$$

(3.1.15) states that the water price or the marginal benefit of the water use must equal the cost of obtaining water from other cities. This provides the optimal rule for the use of water from other cities.

Taking the time derivative of (3.1.10) yields the following:

$$\dot{p}(t) = C_G^g(G(t))\dot{G}(t) + \dot{\lambda}(t) \quad (3.1.16)$$

Substituting in (3.1.6), (3.1.7), and (3.1.10), (3.1.16) becomes:

$$\dot{p}(t) = C_G^g(G(t))\{\tau R + \alpha a(t)\} + r\{p(t) - C^g(G(t))\} \quad (3.1.17)$$

As can be seen, the two terms on the right-hand side of (3.1.17) have opposite signs. So, even though the benefit function is assumed to grow over time, it is unclear whether the price is rising or falling along optimal path.

In steady state, sustainability of ground water implies  $\dot{\lambda}(t) = \dot{G}(t) = \dot{p}(t)$ . The following can be obtained:

$$\lambda^* = -\frac{C_G^g(G^*)g^*}{r} \quad (3.1.18)$$

$$g^* = \tau R + \alpha a^* \quad (3.1.19)$$

$$0 = C_G^g(G^*)\{\tau R + \alpha a^*\} + r\{p^* - C^g(G^*)\} \quad (3.1.20)$$

Given  $p^*$  and  $a^*$ , unique  $G^*$  can be derived from (3.1.20). This is because the derivative with respect to  $G^*$  of the right-hand side of (3.1.20) is unambiguously positive,

$C_{GG}^s(G)\{\tau R + \omega a\} - rC_G^s(G) > 0$ . Note also that, with finite  $\lambda^*$  from (3.1.18), (3.1.8) automatically holds.

If the demand is high enough and growing, which again could come from increasing population, rising income, and growing general economic activities in the area, eventually the aquifer and the lakes will not be able to completely satisfy the city's water needs. Another source of water may be needed. That is when water from other cities comes in.<sup>8</sup> Therefore, (3.1.15) requires that  $p^* = C^b$  in steady state. Moreover, if  $C^b$  is sufficiently high, one has that  $\lambda^* > \frac{c^a}{\omega}$  and water is pumped from the lakes at full capacity in steady state. With  $a^* = \bar{a}$ , it can then be obtained from (3.1.19) that  $g^* = \tau R + \omega \bar{a}$  or ground water outflow equals ground water inflow. Also substituting  $a^* = \bar{a}$  and  $p^* = C^b$  in (3.1.20) gives:

$$0 = C_G^s(G^*)\{\tau R + \omega \bar{a}\} + r\{C^b - C^s(G^*)\} \quad (3.1.21)$$

(3.1.21) is an implicit equation for the unknown  $G^*$ .

If  $\frac{c^a}{\omega}$  is high enough, though not as high as  $C^b$ , no water is pumped from the lakes when the demand is low. Thinking of water from the lakes as artificially recharged ground water, because it has to be pumped to the aquifer, one has that it costs more to produce potable water from artificially recharged ground water than from natural ground water.<sup>9</sup> In this sense, the city resorts to water from the lakes only when needed (i.e. when it is less costly).

Depending on the parameter values and the functional forms chosen, the solution may entail the following stages. Initially, the demand is low so that the marginal benefit of extracting ground water is not that much different from the marginal extraction cost (i.e.  $\lambda(t) = p(t) - C^s(G(t)) \rightarrow 0$ ). Natural ground water alone can satisfy the city's water need. No water is pumped from the lakes. No water is obtained from other cities. The optimal control problem in this period is governed by the system of differential equations below:

<sup>8</sup> This water takes care of the growing water demand in the long run.

<sup>9</sup> Water naturally percolates down to the aquifer.

$$\dot{G}(t) = \tau R - g(t) \quad (3.1.22)$$

$$\dot{p}(t) = C_G^s(G(t))\tau R + r\{p(t) - C^s(G(t))\} \quad (3.1.23)$$

$$\text{where } p(t) = B_1(g(t), t)$$

The demand grows so that the city becomes indifferent between no pumping and pumping any amount within  $\bar{a}$  from the lakes. Nevertheless, the demand is not yet at a point where water from other cities is needed. In this period  $t_{II}$ , one has  $\dot{\lambda}(t_{II}) = 0$ . From (3.1.6),  $g(t_{II}) = -\frac{rC^a}{\omega C_G^s(G(t_{II}))}$  is derived. Substituting into (3.1.10) where  $p(t_{II}) \equiv B_1(g(t_{II}) + b(t_{II}), t_{II})$ ,  $b(t_{II}) = 0$ , and  $\lambda(t_{II}) = \frac{C^a}{\omega}$ , the following can be obtained:

$$B_1\left(-\frac{rC^a}{\omega C_G^s(G(t_{II}))}, t_{II}\right) = C^s(G(t_{II})) + \frac{C^a}{\omega} \quad (3.1.24)$$

(3.1.24) is an implicit equation for  $G(t_{II})$  in period  $t_{II}$  where  $\lambda(t_{II}) = \frac{C^a}{\omega}$ .

The demand grows so that water is always pumped from the lakes at full capacity. Artificially recharged ground water is used alongside natural ground water. However, water from other cities is not yet used as the marginal benefit of consuming water is still below the marginal cost of obtaining water from other cities. The optimal control problem in this period is governed by:

$$\dot{G}(t) = \tau R + \omega \bar{a} - g(t) \quad (3.1.25)$$

$$\dot{p}(t) = C_G^s(G(t))\{\tau R + \omega \bar{a}\} + r\{p(t) - C^s(G(t))\} \quad (3.1.26)$$

$$\text{where } p(t) = B_1(g(t), t)$$

The demand grows so high so that the marginal benefit of consuming water reaches the marginal cost of obtaining water from other cities  $C^b$ . The system reaches steady state. All sources of water are used. Water from other cities supplies part of the demand not yet satisfied by the ground water.

Solving these differential equations requires techniques used in dealing with the boundary value problems. To see clearly how this works and the dynamic behavior of the system, let's look at a numerical example for Ames, Iowa below.

### 3.2 Numerical Example

The city of Ames, Iowa relies on a ground water system for its potable water. The water is harvested from the Ames aquifer (Alluvial<sup>10</sup> and Pleistocene<sup>11</sup>) via 19 ground water wells. The aquifer is about 100 feet or less deep and is regularly recharged from rainfall at an assumed rate of  $\tau = 0.3$ . The city's average annual liquid precipitation is 34.11 inches which is approximately equivalent to 13,000,000 thousand gallons of water.<sup>12</sup> So  $R = 13,000,000$ .

When the demand is high and cannot be satisfied by the natural ground water alone, water can be pumped from Hallett's Quarry,<sup>13</sup> Peterson's Pits, and possibly other surface water sources in the area to spreading basins to artificially recharge the aquifer. Since only about 75% of the pumped water actually reaches the aquifer (Simpkins & Christianson, 2005, p. 25), let  $\omega = 0.75$ . For illustrative purpose, let's assume the constraint on the artificial recharge is 1,000,000 thousand gallons of water per pumping. So  $\bar{a} = 1,000,000$ . According to John R. Dunn (personal communication, April 30, 2007), director of Ames water and pollution control department, the cost of the most recent pumping operation (in the fall of 2000) was approximately \$0.6 per a thousand gallon of pumped water.<sup>14</sup> So  $C^a = 0.6$ .

In addition to the aquifer and the lakes, let's assume the city has an option of having water transported in from other cities at a high cost of \$6 per a thousand gallon of water.<sup>15</sup> So  $C^b = 6$ .

The marginal cost of ground water extraction and the benefit associated with water use are modeled as in Krulce et al. (1997). Specifically, the functional form for the cost function is  $C^g(G(t)) = C_{base} \left( \frac{G_{base}}{G(t)} \right)^n$ , where  $C_{base}$  is the marginal ground water extraction cost

<sup>10</sup> Geological deposits of the current river valley composed of clay, silt, sand, gravel, or similar material deposited by running water

<sup>11</sup> Similar to Alluvial, but more surface sediment covering a prehistoric buried channel formation

<sup>12</sup> See <http://mesonet.agron.iastate.edu/request/coop/fe.phtml> for liquid precipitation data for Ames, Iowa.

<sup>13</sup> Converted into 1,200,000-thousand-gallon Ada Hayden lake in 2004

<sup>14</sup> The costs of pumping 35.38 million gallons of water from Peterson's Pits over 27 days in the fall of 2000 were \$22,105 of labor. Diesel engine/generator set was used. However, it consumed very little fuel (i.e. 2 to 3 gallons per day maximum for diesel fuel).

<sup>15</sup> So far this option has never been used by the city. On the contrary, the city has been providing water to Xenia rural water district. Krulce et al. (1997) uses \$3.00 per a thousand gallon of water as the unit cost of desalination for the city of Oahu, Hawaii.

at the ground water stock  $G_{base}$ .  $n$  determines the rate at which the cost rises to infinity as the stock gets close to exhaustion ( $G(t) = 0$ ). As can be seen,  $C_G^g(G(t)) = -\frac{nC_{base}}{G(t)} \left(\frac{G_{base}}{G(t)}\right)^n < 0$  and  $C_{GG}^g(G(t)) = \frac{n\{n+1\}C_{base}}{G(t)^2} \left(\frac{G_{base}}{G(t)}\right)^n > 0$ . For illustrative purpose, let's assume  $C_{base} = 0.3$ ,  $G_{base} = 15,000,000$ , and  $n = 1$ .

The benefit function is modeled as  $B(g(t) + b(t), t) = \int_0^{g(t)+b(t)} \left(\frac{\alpha e^{\beta t}}{X}\right)^{\frac{1}{\eta}} dX$ , yielding a constant elasticity demand function that grows over time at a constant rate  $\beta$  of the form

$$p(t) = \left(\frac{\alpha e^{\beta t}}{g(t)+b(t)}\right)^{\frac{1}{\eta}}. \quad \eta \text{ is the demand elasticity. Also it can be derived further that}$$

$B_{11}(g(t) + b(t), t) = -\frac{1}{\eta\{g(t)+b(t)\}} \left(\frac{\alpha e^{\beta t}}{g(t)+b(t)}\right)^{\frac{1}{\eta}} < 0$ . For illustrative purpose, let's assume  $\beta = 0.01$  and<sup>16</sup>  $\eta = 0.5$ .  $\alpha$  is chosen to normalize the demand to actual price and quantity data. As of now, the price of potable water in Ames is \$2 per a thousand gallon of water and the water consumption is approximately 2,500,000 thousand gallons per year.<sup>17</sup> Therefore,

$$\alpha = g(0) * p(0)^\eta = 2,500,000 * 2^{0.5} = 3,535,534.$$

Again, following Krulce et al. (1997), the real social discount rate  $r = 0.03$  is used.

**Table 1** summarizes all the parameter values for Ames, Iowa:

**Table 1. Parameter values for Ames, Iowa (growing demand)**

| Parameter  | Value      | Parameter  | Value      |
|------------|------------|------------|------------|
| $\tau$     | 0.3        | $G_{base}$ | 15,000,000 |
| $R$        | 13,000,000 | $n$        | 1          |
| $\omega$   | 0.75       | $\beta$    | 0.01       |
| $\bar{a}$  | 1,000,000  | $\eta$     | 0.5        |
| $C_a$      | 0.6        | $\alpha$   | 3,535,534  |
| $C^b$      | 6          | $r$        | 0.03       |
| $C_{base}$ | 0.3        |            |            |

<sup>16</sup> Water demand elasticity ranges from 0.1 to 0.7 in absolute value. See Olmstead (2009), Martinez-Espineira (2007), Nauges (2003), Renwick, Green, and McCorkle (1998), and Thomas and Syme (1988) for more details.

<sup>17</sup> See <http://www.cityofames.org/WaterWeb/WaterPlant/Home.htm> for more details.

With all the parameter values and the functional forms specified, the system can be solved using Mathematica. In steady state, one has  $p^* = C^b = 6$  and  $a^* = \bar{a} = 1,000,000$ . (3.1.19) yields  $g^* = 4,650,000$ . (3.1.21) gives  $G^* = 11,163,400$ . From (3.1.18),  $\lambda^* = 5.6 > 0.8 = \frac{c^a}{\omega}$  is obtained, ensuring full capacity pumping of water from the lakes in steady state.

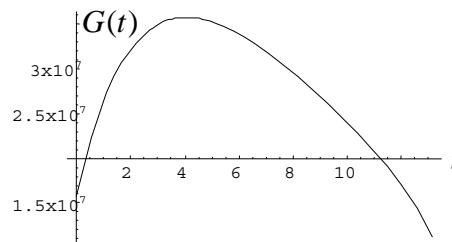
Let's now turn to period  $t_{II}$  where the city is indifferent between no pumping and pumping any amount within  $\bar{a}$  from the lakes. In this case, (3.1.24) gives the optimal ground water stock  $G(t_{II})$  that is increasing in  $t_{II}$ . This results in  $G(t_{II} = 0) = 25,919,800$ . The optimal price  $p(t_{II})$  can then be derived from (3.1.10). Let  $t_{II}^{start}$  be the time when the system enters period  $t_{II}$ . With  $G(t_{II})$ ,  $p(t_{II})$ ,  $G_0$ , (3.1.22), and (3.1.23),  $t_{II}^{start}$  can be obtained using NDSolve in Mathematica.<sup>18</sup> However, if the initial ground water stock  $G_0$  is less than  $G(t_{II} = 0)$ , positive  $t_{II}^{start}$  can never be found. As a result, it is not optimal to start with no artificial recharge ( $a(t) = 0$ ) when the initial ground water stock is small ( $G_0 < G(t_{II} = 0)$ ). Instead, water should be pumped from the lakes at full capacity ( $a(t) = \bar{a}$ ) right from the beginning.

### 3.2.1 Small $G_0 < G(t_{II} = 0)$

According to Zientara (2006), the city's water source capacity (wells and supply capacity) is about 10,500 to 11,000 thousand gallons per day. If this implies, for instance,  $G_0 = 15,700,000$  thousand gallons (four years of supply) and since  $G_0 = 15,700,000 < 25,919,800 = G(t_{II} = 0)$ , it is the case for the city to always pump water from the lakes at full capacity. The social planner chooses  $G(t)$  and  $p(t)$  that evolve as (3.1.25) and (3.1.26) respectively such that the system moves toward steady state.

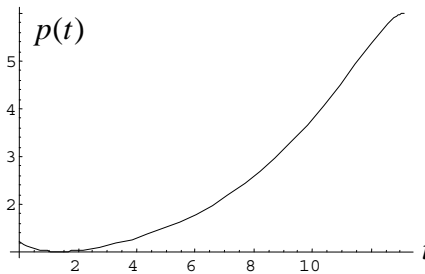
<sup>18</sup> This is a boundary value problem. The solution method requires solving for  $t_{II}^{start}$  such that the solution to the system of differential equations (3.1.22) and (3.1.23) with boundary conditions  $G(t_{II}^{start})$  and  $p(t_{II}^{start})$  results in the initial ground water stock  $G_0$ .

Let  $t_s$  be the time when the system enters steady state. With  $G^* = 11,163,400$ ,  $p^* = 6$ ,  $G_0 = 15,700,000$ , (3.1.25), and (3.1.26), it is found that  $t_s = 131.17$ . Therefore, it would take approximately 131 years before water from other cities is used. During this period, the optimal ground water stock is allowed to first rise to reduce future extraction cost of ground water. The process reverses at some point, as the demand grows, and the optimal stock falls to the steady state level. This is shown in **Figure 1**:



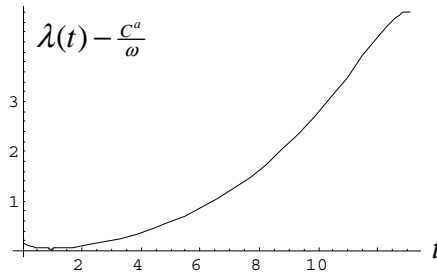
**Figure 1. Optimal stock of ground water when  $G_0$  is small**

The optimal water price, on the other hand, falls at first and then rises to the steady state level  $C^b$ . This can be seen in **Figure 2**:



**Figure 2. Optimal price of water when  $G_0$  is small**

Over time, the marginal user cost is always above the unit cost of water that actually goes down to the aquifer by means of pumping ( $\lambda(t) > \frac{c^a}{\omega} = 0.8$ ), which is why the artificial recharge is done at full capacity in this case. **Figure 3** shows the difference over time between these two numbers ( $\lambda(t) - \frac{c^a}{\omega}$ ). As can be seen, the difference is always positive.

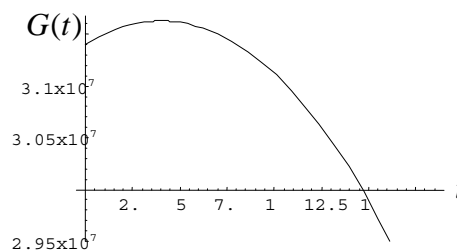


**Figure 3.**  $\lambda(t) - \frac{C^a}{\omega}$  when  $G_0$  is small

### 3.2.2 Large $G_0 > G(t_{II} = 0)$

Conversely, if the initial ground water stock  $G_0$  is greater than  $G(t_{II} = 0)$ , positive  $t_{II}^{start}$  can be found. For illustrative purpose, let the initial ground water stock be 31,400,000 thousand gallons (eight years of supply). Since  $G_0 = 31,400,000 > 25,919,800 = G(t_{II} = 0)$ ,

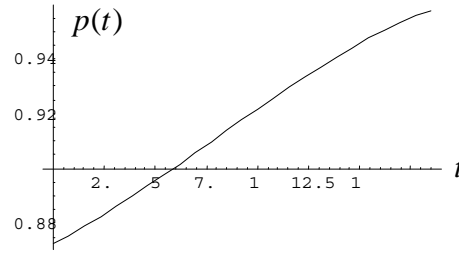
$t_{II}^{start} = 18.5$  is found. Therefore, the period of no artificial recharge ( $a(t) = 0$ ) should last approximately 18 years. During this period, the ground water stock first increases as the natural addition to the ground water stock is greater than the ground water extraction. The process reverses at some point, as the demand grows, and the ground water stock decreases until it equals  $G(t_{II}^{start} = 18.5) = 28,549,700$ . This is shown in **Figure 4**:



**Figure 4.** Ground water stock for period when  $a(t) = 0$  and  $G_0$  is large

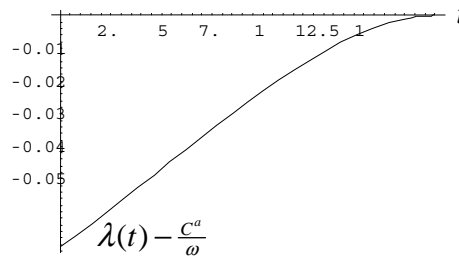
The water price, on the other hand, increases over time from \$0.872 to \$0.958 per a thousand gallon, which can be seen in **Figure 5**:





**Figure 5.** Water price for period when  $a(t) = 0$  and  $G_0$  is large

Moreover, the marginal user cost is always below the unit cost of water that actually goes down to the aquifer by means of pumping in this period ( $\lambda(t) < \frac{c^a}{\omega} = 0.8$ ), which is why no artificial recharge is done. **Figure 6** shows the difference over time between these two numbers ( $\lambda(t) - \frac{c^a}{\omega}$ ). The difference is always negative.

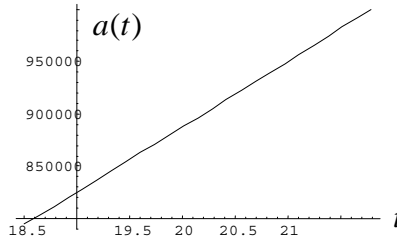


**Figure 6.**  $\lambda(t) - \frac{c^a}{\omega}$  for period when  $a(t) = 0$  and  $G_0$  is large

Once  $t_{II}^{start}$  is reached, the system enters period  $t_{II}$  where  $\lambda(t_{II}) = \frac{c^a}{\omega}$ . In this period, the optimal ground water stock is  $G(t_{II})$  and<sup>19</sup> the optimal price is  $p(t_{II}) = C^g(G(t_{II})) + \frac{c^a}{\omega}$ .  $g(t_{II})$  can then be derived from  $B_1(g(t_{II}), t_{II}) = p(t_{II})$ . The optimal ground water stock evolves as  $\dot{G}(t_{II}) = \pi R + \omega a(t_{II}) - g(t_{II})$ , which results in  $a(t_{II}) = \frac{\dot{G}(t_{II}) - \pi R + g(t_{II})}{\omega}$ . Let  $t_{II}^{end}$  be the time when the system exits period  $t_{II}$ . Setting  $a(t_{II})$  equal  $\bar{a}$ , one can solve for  $t_{II}^{end}$ . In this case,  $a(t_{II})$  is increasing in  $t_{II}$ . For the solution to exist, it must be that  $a(t_{II}^{start}) < \bar{a}$ , which is the case for  $\bar{a} = 1,000,000$  as  $a(t_{II}^{start} = 18.5) = 794,677 < 1,000,000 = \bar{a}$ .

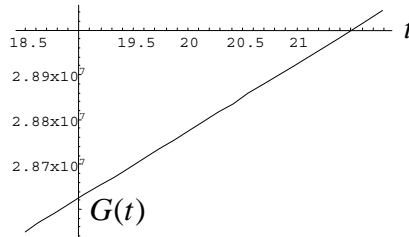
<sup>19</sup> Again, (3.1.24) provides an implicit equation for  $G(t_{II})$ .

$t_{II}^{end} = 21.79$  is then found. Therefore, period  $t_{II}$  where  $\lambda(t_{II}) = \frac{c^a}{\omega}$  would last approximately 3 years. **Figure 7** shows the optimal artificial ground water recharge over this period. As can be seen, the optimal artificial recharge rises over time until it reaches full capacity.



**Figure 7.** Optimal artificial ground water recharge for period  $t_{II}$  when  $G_0$  is large

At the same time, the optimal stock of ground water increases from  $G(t_{II}^{start} = 18.5) = 28,549,700$  to  $G(t_{II}^{end} = 21.79) = 29,043,000$  in this period, which is shown in **Figure 8**:

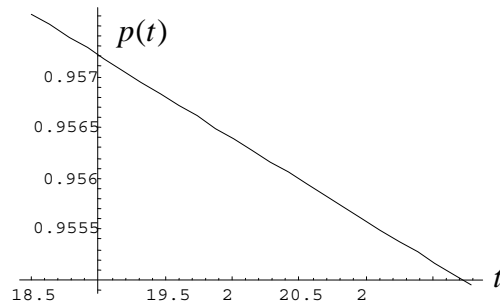


**Figure 8.** Optimal ground water stock for period  $t_{II}$  when  $G_0$  is large

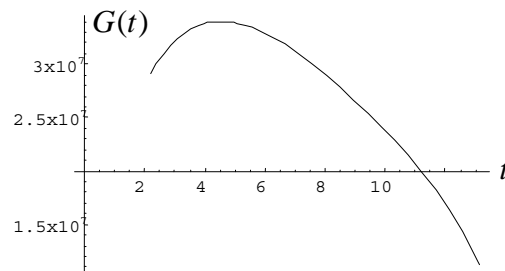
The optimal price, on the other hand, decreases slightly from \$0.958 to \$0.955 per a thousand gallon, which can be seen in **Figure 9**.

Once the system reaches  $t_{II}^{end}$ , water would be pumped from the lakes at full capacity. With  $G^* = 11,163,400$ ,  $p^* = 6$ ,  $G(t_{II}^{end} = 21.79) = 29,043,000$ , (3.1.25), and (3.1.26), it is found that  $t_s = 131.11$ . Therefore, this period of full capacity artificial ground water recharge would last approximately 110 years before steady state is attained. In this period,

the optimal ground water stock rises from  $G(t_{II}^{end} = 21.79) = 29,043,000$  for some time. It then falls to the steady state level  $G^* = 11,163,400$ . This is depicted in **Figure 10**.

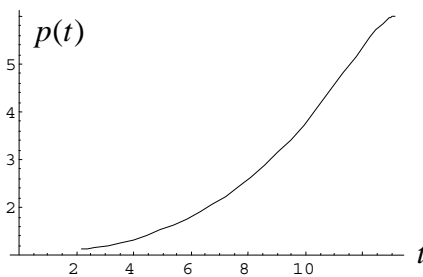


**Figure 9.** Optimal water price for period  $t_{II}$  when  $G_0$  is large



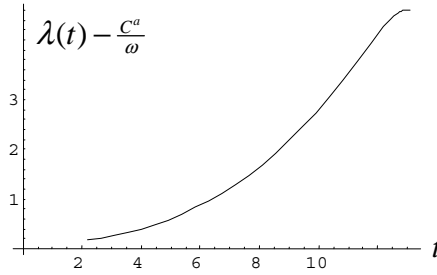
**Figure 10.** Optimal ground water stock for period when  $a(t) = \bar{a}$  and  $G_0$  is large

The optimal water price rises over time until the steady state level  $C^b$  is reached. It then stays there forever. This is shown in **Figure 11**:



**Figure 11.** Optimal water price for period when  $a(t) = \bar{a}$  and  $G_0$  is large

In addition, the marginal user cost is always above the unit cost of water that actually goes down to the aquifer by means of pumping in this period ( $\lambda(t) > \frac{c^a}{\omega} = 0.8$ ), which is why the artificial recharge is done at full capacity. **Figure 12** shows the difference over time between these two numbers ( $\lambda(t) - \frac{c^a}{\omega}$ ). The difference is always positive.



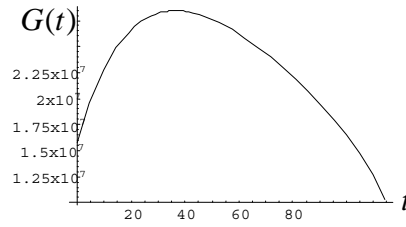
**Figure 12.**  $\lambda(t) - \frac{c^a}{\omega}$  for period when  $a(t) = \bar{a}$  and  $G_0$  is large

### 3.2.3 No Artificial Recharge Allowed at All Times

Note that if the artificial ground water recharge is not allowed,<sup>20</sup> the problem becomes precisely Krulce et al. (1997). The city relies entirely on its natural ground water when the demand is low. Water from other cities is used only when the demand becomes so high such that the price equals the cost of obtaining water from other cities. Setting  $a^* = 0$ , one has from (3.1.20) that the steady state ground water stock is  $G^* = 10,256,300$  and from (3.1.19) that the steady state ground water extraction is  $g^* = 3,900,000$ .

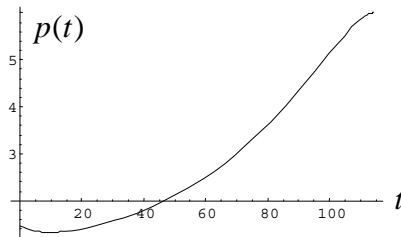
With small initial ground water stock  $G_0 = 15,700,000$ , it would take approximately 114.2 years for the city to start using water from other cities. The optimal ground water stock is first allowed to rise to reduce the future extraction cost. At some point, as the demand grows, the optimal stock falls until it reaches the steady state level. This is shown in **Figure 13**:

<sup>20</sup> Surface water cannot be pumped to the aquifer.



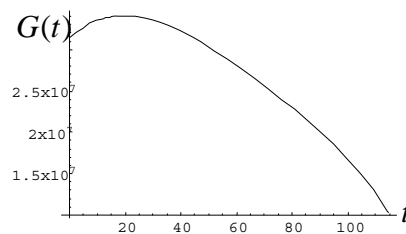
**Figure 13. Optimal ground water stock when no artificial recharge is allowed and  $G_0$  is small**

The optimal price, on the other hand, falls at first and later rises to the steady state level. This is shown in **Figure 14**:



**Figure 14. Optimal water price when no artificial recharge is allowed and  $G_0$  is small**

With large initial ground water stock  $G_0 = 31,400,000$ , it would take approximately 114.5 years for the city to start using water from other cities. **Figure 15** shows the optimal path of the ground water stock in this case.



**Figure 15. Optimal ground water stock when no artificial recharge is allowed and  $G_0$  is large**

The optimal price path is shown in **Figure 16** below:

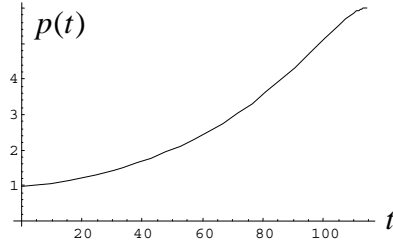


Figure 16. Optimal water price when no artificial recharge is allowed and  $G_0$  is large

As can be seen, it takes longer for costly water from other cities to be used when the artificial recharge of the aquifer is possible.

#### 4 Stochastic Rainfall

This section examines the impact of stochastic rainfall on the intertemporal water use. To focus on water scarcity problem posed by stochastic rainfall, constant water demand is assumed. Faced with increase in rainfall variability, the city may not grow any longer.

Let rainfall be stochastic. If  $RS(t)$  is the amount of rainfall the city has had up until time  $t$  and  $dRS(t)$  follows the Ito's process, the amount of rainfall per period  $dt$  can be expressed as the following:

$$R(t)dt = dRS(t) = Rdt + \Gamma dz, \quad dz = \varepsilon_t \sqrt{dt}, \quad \text{and}^{21} \quad \varepsilon_t \sim N(0,1) \quad (4.1)$$

where  $R$  is the expected instantaneous rainfall and  $\frac{\Gamma^2}{dt}$  is the instantaneous variance. The ground water stock, in this case, changes according to the following differential equation:

$$dG(t) = \tau(Rdt + \Gamma dz) + (\omega a(t) - g(t))dt = (\tau R + \omega a(t) - g(t))dt + \tau \Gamma dz \quad (4.2)$$

As a result, the optimization problem becomes:

$$\max_{g(t), a(t), b(t), G(t)} E_0 \left\{ \int_0^{\infty} e^{-rt} [B(g(t) + b(t)) - C^g(G(t))g(t) - C^a a(t) - C^b b(t)] dt \right\} \quad (4.3)$$

$$\text{subject to } dG(t) = (\tau R + \omega a(t) - g(t))dt + \tau \Gamma dz, \quad dz = \varepsilon_t \sqrt{dt}, \quad \text{and } \varepsilon_t \sim N(0,1)$$

$$a(t) \leq \bar{a}, \quad g(t), a(t), b(t) \geq 0$$

$$G(0) = G_0$$

<sup>21</sup> Normal distribution is widely used in the literature because it is well behaved and mathematically tractable. Central limit theorem provides a theoretical justification for its use.

#### 4.1 Optimal Rules

The Bellman equation of system (4.3) can then be expressed as follows:

$$V(G(t)) = \max_{g(t), a(t), b(t)} \left\{ \left[ B(g(t) + b(t)) - C^s(G(t))g(t) - C^a a(t) - C^b b(t) \right] dt \right. \\ \left. + e^{-rdt} E_t(V(G(t+dt))) \right\} \quad (4.1.1)$$

subject to  $dG(t) = (\tau R + \omega a(t) - g(t))dt + \sigma \Gamma dz$ ,  $dz = \varepsilon_t \sqrt{dt}$ , and  $\varepsilon_t \sim N(0,1)$

$$a(t) \leq \bar{a}$$

$$g(t), a(t), b(t) \geq 0$$

$$G(0) = G_0$$

Applying Taylor's series expansion and Ito's lemma, one has:

$$V(G(t+dt)) = V(G(t)) + V_G(G(t))dG(t) + \frac{1}{2}V_{GG}(G(t))\{dG(t)\}^2 \quad (4.1.2)$$

Substituting in  $dG(t) = (\tau R + \omega a(t) - g(t))dt + \sigma \Gamma dz$  and  $\{dG(t)\}^2 = \varepsilon_t^2 \{\sigma \Gamma\}^2 dt$ , (4.1.2)

becomes:

$$V(G(t+dt)) = \left\{ \begin{aligned} &V(G(t)) + [V_G(G(t))\{\tau R + \omega a(t) - g(t)\} + \frac{1}{2}V_{GG}(G(t))\varepsilon_t^2 \{\sigma \Gamma\}^2] dt \\ &+ V_G(G(t))\sigma \Gamma dz \end{aligned} \right\} \quad (4.1.3)$$

Taking expectation of (4.1.3) and using  $E_t(dz) = 0$  and  $E_t(\varepsilon_t^2) = 1$ , one has the following:

$$E_t(V(G(t+dt))) = V(G(t)) + \left[ \begin{aligned} &V_G(G(t))\{\tau R + \omega a(t) - g(t)\} \\ &+ \frac{1}{2}V_{GG}(G(t))\{\sigma \Gamma\}^2 \end{aligned} \right] dt \quad (4.1.4)$$

Rearranging (4.1.4) gives:

$$\frac{1}{dt} E_t dV(G(t)) = \frac{1}{dt} E_t (V(G(t+dt)) - V(G(t))) = \left[ \begin{aligned} &V_G(G(t))\{\tau R + \omega a(t) - g(t)\} \\ &+ \frac{1}{2}V_{GG}(G(t))\{\sigma \Gamma\}^2 \end{aligned} \right] \quad (4.1.5)$$

Substituting (4.1.4) into (4.1.1) and noting that  $e^{-rdt} \approx (1 - rdt)$  and  $(dt)^2$  goes to zero faster than  $dt$  for an infinitesimally small  $dt$ , it can be obtained that:

$$rV(G(t)) = \max_{g(t), a(t), b(t)} \left\{ \begin{aligned} &B(g(t) + b(t)) - C^s(G(t))g(t) - C^a a(t) - C^b b(t) \\ &+ V_G(G(t))[\tau R + \omega a(t) - g(t)] + \frac{1}{2}V_{GG}(G(t))\{\sigma \Gamma\}^2 \end{aligned} \right\} \quad (4.1.6)$$

subject to  $dG(t) = (\tau R + \omega a(t) - g(t))dt + \sigma \Gamma dz$ ,  $dz = \varepsilon_t \sqrt{dt}$ , and  $\varepsilon_t \sim N(0,1)$

$$a(t) \leq \bar{a}, \quad g(t), a(t), b(t) \geq 0$$

$$G(0) = G_0$$

The first order necessary conditions for an optimal solution are shown below:

$$g(t) : \begin{cases} B_1(g(t) + b(t)) - C^g(G(t)) \\ -V_G(G(t)) \end{cases} \leq 0, g(t) \geq 0, \{B_1(\cdot) - C^g(\cdot) - V_G(\cdot)\}g(t) = 0 \quad (4.1.7)$$

$$b(t) : B_1(g(t) + b(t)) - C^b \leq 0, b(t) \geq 0, \{B_1(\cdot) - C^b\}b(t) = 0 \quad (4.1.8)$$

$$a(t) : -C^a + V_G(G(t))\omega < 0, a(t) = 0 \quad (4.1.9)$$

$$> 0, a(t) = \bar{a}$$

$$= 0, 0 \leq a(t) \leq \bar{a}$$

where  $V_G(G(t))$  is the benefit of having an additional unit of water underground (i.e. marginal user cost of ground water). (4.1.7) and (4.1.8) provide optimal rules for uses of ground water and water from other cities respectively. Water is extracted from each source when the marginal net benefit associated with water use equals zero. (4.1.9) provides optimal rule for the artificial ground water recharge. Since the optimization problem is linear in the amount of artificial recharge, water is pumped to the aquifer when the net benefit of having  $\omega$  additional units of water underground is greater than or equal to zero.

## 4.2 Solutions

Assuming high cost of obtaining water from other cities (i.e. assuming sufficiently high  $C^b$ ), it must be that water is pumped to the aquifer at full capacity  $\bar{a}$ , regardless of whether the ground water is used, whenever water from other cities is used. In addition, the city should extract as much as possible from the less expensive water sources before extracting from the more expensive ones. As a result, five solution cases are feasible. They are as shown in **Figure 17**. At time  $t$ , given the ground water stock  $G(t)$ , the city chooses how much  $g(t)$  to extract from the aquifer, how much  $a(t)$  to pump to the aquifer, and how much  $b(t)$  to obtain from other cities.

For **Case I**, **Case II**, and **Case III**, the city relies entirely on ground water (i.e. using no water from other cities). So it must be from (4.1.7) that the marginal benefit associated with water use equals the marginal cost of using ground water:

$$B_1(g(t)) = C^g(G(t)) + V_G(G(t)) \quad (4.2.1)$$



and from (4.1.8) that the marginal benefit associated with water use is less than the marginal cost of obtaining water from other cities:

$$B_1(g(t)) < C^b \quad (4.2.2)$$

Given the current ground water stock  $G(t)$  and that the value function  $V(G(t))$  is known, the city knows exactly from (4.2.1) how much ground water to extract:

$$g(t) = B_1^{-1}(C^g(G(t)) + V_G(G(t))) \quad (4.2.3)$$

As can be seen, the amount of ground water extracted at time  $t$  does not depend on current rainfall.

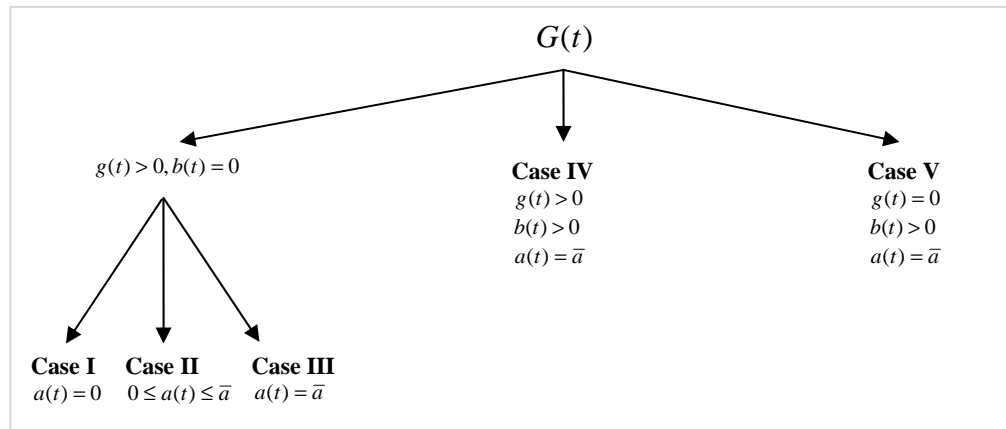


Figure 17. Five possible solution cases under stochastic rainfall and sufficiently high  $C^b$  assumptions

In this sense, no ground water conservation is practiced in a time of drought in **Case I**, **Case II**, and **Case III**. For the city to be able to rely entirely on ground water at time  $t$ , the current ground water stock  $G(t)$  must be sufficiently large. The large stock of ground water makes it possible for the city to extract without having to worry about current rainfall.

**Case I:**  $g(t) > 0, b(t) = 0, a(t) = 0$

Since no water is pumped to the aquifer, it must be from (4.1.9) that the benefit of having an additional unit of water underground is less than the cost:

$$V_G(G(t)) < \frac{c^a}{\omega} \quad (4.2.4)$$

Substituting (4.2.3),  $b(t) = 0$ , and  $a(t) = 0$  into (4.1.6) gives the following:

$$rV(G(t)) = \left\{ \begin{array}{l} B(B_1^{-1}(C^g(G(t)) + V_G(G(t)))) - C^g(G(t))B_1^{-1} \left( \begin{array}{l} C^g(G(t)) \\ + V_G(G(t)) \end{array} \right) \\ + V_G(G(t))[\tau R - B_1^{-1}(C^g(G(t)) + V_G(G(t)))] + \frac{1}{2}V_{GG}(G(t))[\tau \Gamma]^2 \end{array} \right\} \quad (4.2.5)$$

(4.2.5) is a second-order differential equation which can be solved for the value function  $V^{Case1}(G(t))$ . Using  $V_G^{Case1}(G(t))$  in (4.2.3) gives the optimal ground water extraction:

$$g^{Case1}(t) = B_1^{-1}(C^g(G(t)) + V_G^{Case1}(G(t))) \quad (4.2.6)$$

where the ground water stock  $G(t)$  must be sufficiently large so that the city resorts to neither artificial ground water recharge nor water from other cities (i.e.  $G(t)$  must be such that  $V_G^{Case1}(G(t)) < \frac{C^a}{\omega}$  and  $B_1(g^{Case1}(t)) < C_s^b$  hold). Although current rainfall has no impact on the amount of ground water extracted at time  $t$ , it does determine the level of ground water stock at the subsequent time  $t + dt$ . This is because the optimal ground water stock evolves according to  $dG(t) = (\tau R - g^{Case1}(t))dt + \tau \Gamma dz$  in this case.

When current rainfall is sufficiently large (i.e.  $(R + \Gamma \frac{dz}{dt}) > (\frac{g^{Case1}(t)}{\tau})$ ) so that the natural recharge to the aquifer is greater than the amount of water extracted from the aquifer at time  $t$ , the ground water stock at the subsequent time  $t + dt$  must be larger than the current ground water stock (i.e.  $G(t + dt) > G(t)$ ). Unless there is a reason to believe that the marginal user cost of ground water is increasing in the ground water stock, the larger ground water stock at time  $t + dt$  must also be sufficiently large so that the city resorts to neither artificial ground water recharge nor water from other cities at time  $t + dt$  (i.e.  $G(t + dt)$  must be such that  $V_G^{Case1}(G(t + dt)) < \frac{C^a}{\omega}$  and  $B_1(g^{Case1}(t + dt)) < C_s^b$  hold). Therefore, it is optimal for the city to stay in **Case I** at time  $t + dt$ .

When current rainfall is sufficiently small (i.e.  $(R + \Gamma \frac{dz}{dt}) < (\frac{g^{Case1}(t)}{\tau})$ ) so that the natural recharge to the aquifer is smaller than the amount of water extracted from the aquifer at time  $t$ , the ground water stock at the subsequent time  $t + dt$  must be smaller than the current ground water stock (i.e.  $G(t + dt) < G(t)$ ). Assuming the marginal user cost of ground water is decreasing in the ground water stock (i.e. assuming  $V_{GG}^{Case1}(G(t)) < 0$ ), the smaller ground water stock at time  $t + dt$  may not be sufficiently large to keep the city from resorting to

other more expensive water sources at time  $t + dt$  (i.e.  $G(t + dt)$  may not be such that  $V_G^{CaseI}(G(t + dt)) < \frac{c^a}{\omega}$  and  $B_1(g^{CaseI}(t + dt)) < C_s^b$  hold). Therefore, **Case I** may no longer be optimal at time  $t + dt$ .

- If current rainfall is such that the ground water stock at time  $t + dt$  equals<sup>22</sup>  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left(\frac{g^{CaseI}(t)}{\tau}\right) + \left(\frac{G^{CaseII} - G(t)}{dt}\right) < \left(\frac{g^{CaseI}(t)}{\tau}\right)$ ), the city should switch to **Case II** where<sup>23</sup> it is optimal to pump some water to the aquifer in addition to using the natural ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  is smaller than  $G^{CaseII}$  but larger than<sup>24</sup>  $G^{CaseIV}$  (i.e.  $\left(\frac{g^{CaseI}(t)}{\tau}\right) + \left(\frac{G^{CaseIV} - G(t)}{dt}\right) < (R + \Gamma \frac{dz}{dt}) < \left(\frac{g^{CaseI}(t)}{\tau}\right) + \left(\frac{G^{CaseII} - G(t)}{dt}\right)$ ), the city should switch to **Case III** where<sup>25</sup> it is optimal to pump water to the aquifer at full capacity  $\bar{a}$  in addition to using the natural ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left(\frac{g^{CaseI}(t)}{\tau}\right) + \left(\frac{G^{CaseIV} - G(t)}{dt}\right)$ ), the city should switch to **Case IV** where<sup>26</sup> it is optimal to obtain water from other cities at the cost of  $C^b$  and to pump water to the aquifer at full capacity  $\bar{a}$  in addition to using the natural ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  is smaller than  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) < \left(\frac{g^{CaseI}(t)}{\tau}\right) + \left(\frac{G^{CaseIV} - G(t)}{dt}\right)$ ), the city should switch to **Case V** where<sup>27</sup> it is optimal to rely entirely on water from other cities (i.e. using no ground water) while letting the aquifer recharge naturally and artificially.

**Case II:**  $g(t) > 0, b(t) = 0, 0 \leq a(t) \leq \bar{a}$

Since some water is pumped to the aquifer, it must be from (4.1.9) that the benefit of having an additional unit of water underground equals the cost:

<sup>22</sup> See **Case II** for definition of  $G^{CaseII}$ .

<sup>23</sup> See **Case II** for more details.

<sup>24</sup> See **Case IV** for definition of  $G^{CaseIV}$ .

<sup>25</sup> See **Case III** for more details.

<sup>26</sup> See **Case IV** for more details.

<sup>27</sup> See **Case V** for more details.

$$V_G(G(t)) = \frac{C^a}{\omega} \quad (4.2.7)$$

Differentiating (4.1.6) with respect to  $G(t)$ , invoking the envelope theorem, and applying (4.1.5) yields the following:

$$rV_G(G(t)) = -C_G^g(G(t))g(t) + \frac{1}{dt} E_t dV_G(G(t)) \quad (4.2.8)$$

It can then be derived from (4.2.7) and (4.2.8) that:

$$g(t) = -\frac{rC^a}{\omega C_G^g(G(t))} \quad (4.2.9)$$

Substituting (4.2.9) and (4.2.7) in (4.2.1) gives:

$$B_1 \left( -\frac{rC^a}{\omega C_G^g(G^{CaseII})} \right) = C^g(G^{CaseII}) + \frac{C^a}{\omega} \quad (4.2.10)$$

(4.2.10) is an implicit equation for  $G^{CaseII}$ .  $G^{CaseII}$  is the ground water stock that must be maintained so that the benefit of having an additional unit of water underground equals the cost. It is then obtained from (4.2.9) that the ground water extraction must also be constant, i.e.  $g^{CaseII} = -\frac{rC^a}{\omega C_G^g(G^{CaseII})}$ . As a result, the artificial ground water recharge must be allowed to

change according to current rainfall to keep the ground water stock at  $G^{CaseII}$ . In other words, water conservation is not needed in a time of drought as more artificial ground water recharge would be done. Setting  $dG(t)$  equal to zero yields  $a^{CaseII}(t) = \left( \frac{g^{CaseII} - \tau R}{\omega} \right) - \left( \frac{d}{\omega} \right) \frac{dz}{dt}$ .

However, for  $0 \leq a^{CaseII}(t) \leq \bar{a}$  to hold, current rainfall must fall within the below range:

$$\left( \frac{g^{CaseII} - \omega \bar{a}}{\tau} \right) \leq \left( R + \Gamma \frac{dz}{dt} \right) \leq \left( \frac{g^{CaseII}}{\tau} \right) \quad (4.2.11)$$

When current rainfall falls above (4.2.11) range (i.e.  $(R + \Gamma \frac{dz}{dt}) > \left( \frac{g^{CaseII}}{\tau} \right)$ ), the natural recharge to the aquifer is greater than the amount  $g^{CaseII}$  extracted from the aquifer. So  $a(t) < 0$  is required to keep the ground water stock at  $G^{CaseII}$ . But because  $a(t)$  cannot be lower than zero, the ground water stock at the subsequent time  $t + dt$  must be larger than  $G^{CaseII}$ . The larger ground water stock eliminates the need to pump water to the aquifer at time  $t + dt$ . Therefore, the city should switch to **Case I** where it is optimal to rely entirely on the natural ground water.

When current rainfall falls below (4.2.11) range (i.e.  $(R + \Gamma \frac{dz}{dt}) < \left(\frac{g^{CaseII} - \alpha \bar{a}}{\tau}\right)$ ), the natural recharge to the aquifer is smaller than the amount  $g^{CaseII}$  extracted from the aquifer. So  $a(t) > \bar{a}$  is required to keep the ground water stock at  $G^{CaseII}$ . But because  $a(t)$  can only be at most  $\bar{a}$  in this case, the ground water stock at the subsequent time  $t + dt$  must be smaller than  $G^{CaseII}$ . The smaller ground water stock raises the marginal user cost of ground water and may generate the need for other more expensive water sources at time  $t + dt$ . As a result, **Case II** will no longer be optimal at time  $t + dt$ .

- If current rainfall is such that the ground water stock at time  $t + dt$  is smaller than  $G^{CaseII}$  but larger than  $G^{CaseIV}$  (i.e.  $\left(\frac{g^{CaseII} - \alpha \bar{a}}{\tau}\right) + \left(\frac{G^{CaseIV} - G^{CaseII}}{\alpha dt}\right) < (R + \Gamma \frac{dz}{dt}) < \left(\frac{g^{CaseII} - \alpha \bar{a}}{\tau}\right)$ ), the city should switch to **Case III** where it is optimal to pump water to the aquifer at full capacity  $\bar{a}$  in addition to using the natural ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left(\frac{g^{CaseII} - \alpha \bar{a}}{\tau}\right) + \left(\frac{G^{CaseIV} - G^{CaseII}}{\alpha dt}\right)$ ), the city should switch to **Case IV** where it is optimal to obtain water from other cities at the cost of  $C^b$  in addition to using the natural and artificially recharged ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  is smaller than  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) < \left(\frac{g^{CaseII} - \alpha \bar{a}}{\tau}\right) + \left(\frac{G^{CaseIV} - G^{CaseII}}{\alpha dt}\right)$ ), the city should switch to **Case V** where it is optimal to rely entirely on water from other cities (i.e. using no ground water) while letting the aquifer recharge naturally and artificially.

**Case III:**  $g(t) > 0, b(t) = 0, a(t) = \bar{a}$

Since water is pumped to the aquifer at full capacity  $\bar{a}$ , it must be from (4.1.9) that the benefit of having an additional unit of water underground is greater than the cost:

$$V_G(G(t)) > \frac{C^a}{\omega} \quad (4.2.12)$$

Substituting (4.2.3),  $b(t) = 0$ , and  $a(t) = \bar{a}$  into (4.1.6) gives the following:

$$rV(G(t)) = \left\{ \begin{array}{l} B(B_1^{-1}(C^g(G(t)) + V_G(G(t)))) - C^g(G(t))B_1^{-1}(C^g(G(t)) + V_G(G(t))) \\ -C^a\bar{a} + V_G(G(t))[\tau R + \omega\bar{a} - B_1^{-1}(C^g(G(t)) + V_G(G(t)))] \\ \frac{1}{2}V_{GG}(G(t))[\tau\Gamma]^2 \end{array} \right\} \quad (4.2.13)$$

(4.2.13) is a second-order differential equation which can be solved for the value function  $V^{CaseIII}(G(t))$ . Using  $V_G^{CaseIII}(G(t))$  in (4.2.3) gives the optimal ground water extraction:

$$g^{CaseIII}(t) = B_1^{-1}(C^g(G(t)) + V_G^{CaseIII}(G(t))) \quad (4.2.14)$$

where the ground water stock  $G(t)$  must be small for the city to pump water to the aquifer at full capacity  $\bar{a}$ , though not sufficiently small for the city to obtain water from other cities yet (i.e.  $G(t)$  must be such that  $V_G^{CaseIII}(G(t)) > \frac{c^a}{\omega}$  and  $B_1(g^{CaseIII}(t)) < C^b$  hold). Although current rainfall has no impact on the amount of ground water extracted at time  $t$ , it does determine the level of ground water stock at the subsequent time  $t + dt$ . This is because the optimal ground water stock evolves according to  $dG(t) = (\tau R + \omega\bar{a} - g^{CaseIII}(t))dt + \tau\Gamma dz$  in this case.

When current rainfall is sufficiently large (i.e.  $(R + \Gamma \frac{dz}{dt}) > (\frac{g^{CaseIII}(t) - \omega\bar{a}}{\tau})$ ) so that the natural and artificial recharge to the aquifer is greater than the amount of water extracted from the aquifer, the ground water stock at the subsequent time  $t + dt$  must be larger than the current ground water stock (i.e.  $G(t + dt) > G(t)$ ). Assuming the marginal user cost of ground water is decreasing in the ground water stock (i.e. assuming  $V_{GG}^{CaseIII}(G(t)) < 0$ ), the larger ground water stock may reduce or eliminate the need to pump water to the aquifer at time  $t + dt$  (i.e.  $G(t + dt)$  may not be such that  $V_G^{CaseIII}(G(t + dt)) > \frac{c^a}{\omega}$  holds). As a result, **Case III** may no longer be optimal at time  $t + dt$ .

- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = (\frac{g^{CaseIII}(t) - \omega\bar{a}}{\tau}) + (\frac{G^{CaseII} - G(t)}{\tau dt}) > (\frac{g^{CaseIII}(t) - \omega\bar{a}}{\tau})$ ), the city should switch to **Case II** where it is optimal to pump some water to the aquifer in addition to using the natural ground water.

- If current rainfall is such that the ground water stock at time  $t + dt$  is larger than  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) > \left( \frac{g^{CaseIII}(t) - a\bar{a}}{\tau} \right) + \left( \frac{G^{CaseII} - G(t)}{adt} \right)$ ), the city should switch to **Case I** where it is optimal to rely entirely on the natural ground water.

When current rainfall is sufficiently small (i.e.  $(R + \Gamma \frac{dz}{dt}) < \left( \frac{g^{CaseIII}(t) - a\bar{a}}{\tau} \right)$ ) so that the natural and artificial recharge to the aquifer is smaller than the amount of water extracted from the aquifer, the ground water stock at the subsequent time  $t + dt$  must be smaller than the current ground water stock (i.e.  $G(t + dt) < G(t)$ ). Assuming the marginal user cost of ground water is decreasing in the ground water stock (i.e. assuming  $V_{GG}^{CaseIII}(G(t)) < 0$ ), the smaller ground water stock may generate the need for water from other cities at time  $t + dt$  (i.e.  $G(t + dt)$  may not be such that  $B_1(g^{CaseIII}(t + dt)) < C_s^b$  holds). As a result, **Case III** may no longer be optimal at time  $t + dt$ .

- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left( \frac{g^{CaseIII}(t) - a\bar{a}}{\tau} \right) + \left( \frac{G^{CaseIV} - G(t)}{adt} \right) < \left( \frac{g^{CaseIII}(t) - a\bar{a}}{\tau} \right)$ ), the city should switch to **Case IV** where it is optimal to obtain water from other cities at the cost of  $C^b$  in addition to using the natural and artificially recharged ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  is smaller than  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) < \left( \frac{g^{CaseIII}(t) - a\bar{a}}{\tau} \right) + \left( \frac{G^{CaseIV} - G(t)}{adt} \right)$ ), the city should switch to **Case V** where it is optimal to rely entirely on water from other cities (i.e. using no ground water) while letting the aquifer recharge naturally and artificially.

**Case IV:**  $g(t) > 0, b(t) > 0, a(t) = \bar{a}$

The city relies on both ground water and water from other cities. So it must be from (4.1.7) that the marginal benefit associated with water use equals the marginal cost of using ground water:

$$B_1(g(t) + b(t)) = C^g(G(t)) + V_G(G(t)) \quad (4.2.15)$$

and from (4.1.8) that the marginal benefit associated with water use equals the marginal cost of obtaining water from other cities:

$$B_1(g(t) + b(t)) = C^b \quad (4.2.16)$$

It is then derived from (4.2.15) and (4.2.16) that the marginal cost of using ground water must equal the marginal cost of obtaining water from other cities for both ground water and water from other cities to be used:

$$C^g(G(t)) + V_G(G(t)) = C^b \quad (4.2.17)$$

Assuming high cost of obtaining water from other cities (i.e. assuming sufficiently high  $C^b$ ), (4.2.17) implies that the benefit of having an additional unit of water underground is greater than the cost:

$$V_G(G(t)) = C^b - C^g(G(t)) > \frac{C^a}{\omega} \quad (4.2.18)$$

This is why water is pumped to the aquifer at full capacity  $\bar{a}$  whenever water from other cities is used. Again, the city should extract as much as it can from the less expensive water sources before extracting from the more expensive ones.

Differentiating (4.1.6) with respect to  $G(t)$ , invoking the envelope theorem, and applying (4.1.5) yields the following:

$$rV_G(G(t)) = -C_G^g(G(t))g(t) + \frac{1}{dt} E_i dV_G(G(t)) \quad (4.2.19)$$

It can then be derived from (4.2.17) and (4.2.19) that:

$$0 = C_G^g(G^{CaseIV}) \{ \tau R + \omega \bar{a} \} + r \{ C^b - C^g(G^{CaseIV}) \} \quad (4.2.20)$$

(4.2.20) is an implicit equation for  $G^{CaseIV}$ .  $G^{CaseIV}$  is the level of ground water stock that must be maintained so that the marginal cost of using ground water in this case equals the marginal cost  $C^b$  of obtaining water from other cities. As a result, the ground water extraction must be allowed to change according to current rainfall to keep the ground water stock at  $G^{CaseIV}$ . In this sense, water conservation is practiced in a time of drought in **Case IV**. Setting  $dG(t)$  equal to zero yields  $g^{CaseIV}(t) = \tau(R + \Gamma \frac{dz}{dt}) + \omega \bar{a}$  (i.e. the amount of water extracted from the aquifer must equal the natural and artificial recharge to the aquifer). It then follows from (4.2.16) that  $b^{CaseIV}(t) = B_1^{-1}(C^b) - g^{CaseIV}(t)$ . Water from other cities takes care of the excess demand not yet satisfied by the ground water so that the marginal benefit



associated with water use equals the marginal cost  $C^b$  of obtaining water from other cities. However, for  $b^{CaseIV}(t) > 0$  to hold, current rainfall must fall within the below range:

$$(R + \Gamma \frac{dz}{dt}) < \left( \frac{B_1^{-1}(C^b) - \alpha \bar{a}}{\tau} \right) \quad (4.2.21)$$

Since water from other cities takes care of the excess demand in a time of drought, the city will never switch to **Case V** at time  $t + dt$  in this case.

When current rainfall falls outside (4.2.21) range (i.e.  $(R + \Gamma \frac{dz}{dt}) > \left( \frac{B_1^{-1}(C^b) - \alpha \bar{a}}{\tau} \right)$ ), the ground water extraction  $g^{CaseIV}(t)$  is greater than  $B_1^{-1}(C^b)$ . So  $b(t) < 0$  is required to keep the marginal benefit associated with water use equal to the marginal cost  $C^b$  of obtaining water from other cities (i.e.  $b(t) < 0$  is required so that  $B_1(g^{CaseIV}(t) + b(t)) = C^b$  holds). But because  $b(t)$  cannot be lower than zero,  $B_1(g(t) + b(t)) = C^b$  would no longer hold if the ground water extraction was kept at  $g^{CaseIV}(t)$ . As a result, the city should extract  $g^{CaseIVA}(t) = B_1^{-1}(C^b)$  in this case. Since by construction the amount  $B_1^{-1}(C^b)$  extracted from the aquifer is smaller than the natural and artificial recharge  $\tau(R + \Gamma \frac{dz}{dt}) + \alpha \bar{a}$  to the aquifer, the ground water stock at the subsequent time  $t + dt$  must be larger than  $G^{CaseIV}$ . The larger ground water stock reduces or eliminates the need for other more expensive water sources at time  $t + dt$ . As a result, **Case IV** will no longer be optimal at time  $t + dt$ .

- If current rainfall is such that the ground water stock at time  $t + dt$  is larger than  $G^{CaseIV}$  but smaller than  $G^{CaseII}$  (i.e.  $\left( \frac{B_1^{-1}(C^b) - \alpha \bar{a}}{\tau} \right) < (R + \Gamma \frac{dz}{dt}) < \left( \frac{B_1^{-1}(C^b) - \alpha \bar{a}}{\tau} \right) + \left( \frac{G^{CaseII} - G^{CaseIV}}{\Delta t} \right)$ ), the city should switch to **Case III** where it is optimal to pump water to the aquifer at full capacity  $\bar{a}$  in addition to using the natural ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left( \frac{B_1^{-1}(C^b) - \alpha \bar{a}}{\tau} \right) + \left( \frac{G^{CaseII} - G^{CaseIV}}{\Delta t} \right)$ ), the city should switch to **Case II** where it is optimal to extract ground water and pump some water to the aquifer.
- If current rainfall is such that the ground water stock at time  $t + dt$  is larger than  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) > \left( \frac{B_1^{-1}(C^b) - \alpha \bar{a}}{\tau} \right) + \left( \frac{G^{CaseII} - G^{CaseIV}}{\Delta t} \right)$ ), the city should switch to **Case I** where it is optimal to rely entirely on the natural ground water.

**Case V:**  $g(t) = 0, b(t) > 0, a(t) = \bar{a}$

The city relies entirely on water from other cities as the ground water stock becomes significantly low to allow for the ground water recharge. So it must be from (4.1.7) that the marginal benefit associated with water use is less than the marginal cost of using ground water:

$$B_1(b(t)) < C^g(G(t)) + V_G(G(t)) \quad (4.2.22)$$

and from (4.1.8) that the marginal benefit associated with water use equals the marginal cost  $C^b$  of obtaining water from other cities:

$$B_1(b(t)) = C^b \quad (4.2.23)$$

implying  $b^{CaseV} = B_1^{-1}(C^b)$  must be obtained from other cities in this case. As can be seen, water from other cities takes care of all the water demand. It is then derived from (4.2.22) and (4.2.23) that the marginal cost of using ground water must be greater than the marginal cost  $C^b$  of obtaining water from other cities for water from other cities alone to be used:

$$C^g(G(t)) + V_G(G(t)) > C^b \quad (4.2.24)$$

Assuming high cost of obtaining water from other cities (i.e. assuming sufficiently high  $C^b$ ), (4.2.24) implies that the benefit of having an additional unit of water underground is greater than the cost:

$$V_G(G(t)) > C^b - C^g(G(t)) > \frac{C^a}{\omega} \quad (4.2.25)$$

Water is pumped to the aquifer at full capacity  $\bar{a}$  to expedite the ground water recharge.

Since no water is extracted from the aquifer, the ground water stock at the subsequent time  $t + dt$  must be larger than the current ground water stock (i.e.  $G(t + dt) > G(t)$ ).

Assuming the marginal user cost of ground water is decreasing in the ground water stock (i.e. assuming  $V_{GG}(G(t)) < 0$ ), the larger ground water stock may reduce or eliminate the need for other more expensive water sources at time  $t + dt$  (i.e.  $G(t + dt)$  may not be such that  $C^g(G(t + dt)) + V_G(G(t + dt)) > C^b$ ). As a result, **Case V** may no longer be optimal at time  $t + dt$ .

- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseIV}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left(\frac{G^{CaseIV} - G(t)}{\alpha dt}\right) - \left(\frac{\alpha \bar{x}}{\tau}\right)$ ), the city should switch to **Case IV** where it is optimal to obtain water from other cities at the cost of  $C^b$  in addition to using the natural and artificially recharged ground water.
- If current rainfall is such that the ground water stock at time  $t + dt$  is larger than  $G^{CaseIV}$  but smaller than  $G^{CaseII}$  (i.e.  $\left(\frac{G^{CaseIV} - G(t)}{\alpha dt}\right) - \left(\frac{\alpha \bar{x}}{\tau}\right) < (R + \Gamma \frac{dz}{dt}) < \left(\frac{G^{CaseII} - G(t)}{\alpha dt}\right) - \left(\frac{\alpha \bar{x}}{\tau}\right)$ ), the city should switch to **Case III** where it is optimal to extract ground water and pump water to the aquifer at full capacity  $\bar{a}$ .
- If current rainfall is such that the ground water stock at time  $t + dt$  equals  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) = \left(\frac{G^{CaseII} - G(t)}{\alpha dt}\right) - \left(\frac{\alpha \bar{x}}{\tau}\right)$ ), the city should switch to **Case II** where it is optimal to extract ground water and pump some water to the aquifer.
- If current rainfall is such that the ground water stock at time  $t + dt$  is larger than  $G^{CaseII}$  (i.e.  $(R + \Gamma \frac{dz}{dt}) > \left(\frac{G^{CaseII} - G(t)}{\alpha dt}\right) - \left(\frac{\alpha \bar{x}}{\tau}\right)$ ), the city should switch to **Case I** where it is optimal to rely entirely on the natural ground water.

### 4.3 Numerical Example

This section provides a numerical example for Ames, Iowa under stochastic rainfall. The needed parameter values and functional forms are as specified in section 3.2. However,  $\beta = 0$  is used because of the assumed constant water demand. To ease the computations of the value functions  $V^{CaseIII}(G(t))$  from (4.2.13) and  $V^{CaseI}(G(t))$  from (4.2.5),  $\eta = 1$  is used.<sup>28</sup> The benefit function can then be simplified to  $B(g(t) + b(t)) = \alpha \ln(X) \Big|_{\delta}^{g(t)+b(t)}$  where  $\delta$  is a small number close to zero. Since the annual rainfall is assumed to follow the Ito's process, one also needs to specify the variance parameter  $\Gamma$ . According to the city's liquid precipitation data obtained from Iowa Environmental Mesonet,<sup>29</sup>  $\Gamma = 2,800,000$  is used.

**Table 2** summarizes all the parameter values used in this section:

<sup>28</sup> Again,  $\eta$  is the demand elasticity.

<sup>29</sup> See <http://mesonet.agron.iastate.edu/request/coop/fe.phtml> for more details.

**Table 2. Parameter values for Ames, Iowa (stochastic rainfall)**

| Parameter | Value      | Parameter  | Value      |
|-----------|------------|------------|------------|
| $\tau$    | 0.3        | $C_{base}$ | 0.3        |
| $R$       | 13,000,000 | $G_{base}$ | 15,000,000 |
| $\Gamma$  | 2,800,000  | $n$        | 1          |
| $\omega$  | 0.75       | $\beta$    | 0          |
| $\bar{a}$ | 1,000,000  | $\eta$     | 1          |
| $C_a$     | 0.6        | $\alpha$   | 3,535,534  |
| $C^b$     | 6          | $r$        | 0.03       |

With all the parameter values and the functional forms specified, the system can be solved using Mathematica.

**Case II:**  $g(t) > 0, b(t) = 0, 0 \leq a(t) \leq \bar{a}$

From (4.2.10), the ground water stock must be maintained at  $G^{Casell} = 26,110,700$ . From (4.2.9),  $g^{Casell} = 3,636,100$  must be extracted from the aquifer. It then follows that  $a^{Casell}(t) = \left\{ \frac{g^{Casell}}{\omega} \right\} - \left\{ \frac{\tau}{\omega} R(t) \right\} = 4,848,130 - 0.4R(t)$  must be pumped to the aquifer. From (4.2.11), current rainfall must fall within the range  $9,620,320 \leq R(t) \leq 12,120,300$  for  $0 \leq a^{Casell}(t) \leq \bar{a}$  to hold.

- If current rainfall is  $R(t) > 12,120,300$ , the city should pump no water to the aquifer at time  $t$  and should switch to **Case I** at time  $t + 1$ .
- If current rainfall is  $0 < R(t) < 9,620,320$ , the city should pump water to the aquifer at full capacity  $\bar{a} = 1,000,000$  at time  $t$  and should switch to **Case III** at time  $t + 1$ .
- As current rainfall cannot be negative, the city will never switch to **Case IV** nor **Case V** at time  $t + 1$ . With the assumed parameter values and functional forms above, water from other cities is not needed in a time of drought as natural and artificially recharged ground water can completely satisfy the water demand.

**Case IV:**  $g(t) > 0, b(t) > 0, a(t) = \bar{a}$

From (4.2.20), the ground water stock must be maintained at  $G^{CaseIV} = 11,163,400$ . It then follows that  $g^{CaseIV}(t) = \tau R(t) + \omega \bar{a} = 0.3R(t) + 750,000$  must be extracted from the aquifer and  $b^{CaseIV}(t) = B_1^{-1}(C^b) - g^{CaseIV} = -160,744 - 0.3R(t)$  must be obtained from other cities. However, because  $b(t)$  cannot be lower than zero,  $B_1(g(t) + b(t)) = C^b$  would no longer hold if the ground water extraction was kept at  $g^{CaseIV}(t) = 0.3R(t) + 750,000$ . As a result, the city should extract from the aquifer  $g^{CaseIVA}(t) = B_1^{-1}(C^b) = 589,256$  in this case. Since the amount of water entering the aquifer by means of pumping  $\omega \bar{a} = 750,000$  can completely satisfy the water demand at the price  $C^b$ , there is no need for the city to resort to costly water from other cities when the current ground water stock is  $G^{CaseIV} = 11,163,400$  regardless of current rainfall. The ground water stock is then rising.

- If  $0 < R(t) < 49,288,300$ , the city should switch to Case III at time  $t + 1$ .
- If  $R(t) = 49,288,300$ , the city should switch to Case II at time  $t + 1$ .
- If  $R(t) > 49,288,300$ , the city should switch to Case I at time  $t + 1$ .
- According to the city's liquid precipitation data obtained<sup>30</sup>, the maximum annual rainfall over the past 114 years is approximately 22,300,000 thousand gallons of water. As a result, it may be unlikely for the city to switch to Case II or Case I at time  $t + 1$  when the current ground water stock is  $G^{CaseIV} = 11,163,400$ .

**Case V:**  $g(t) = 0, b(t) > 0, a(t) = \bar{a}$

For **Case V** to be optimal, the current ground water stock must be such that  $G(t) < 11,163,400 = G^{CaseIV}$ . From (4.2.23),  $b^{CaseV} = 589,256$  must be obtained from other cities. Since no water is extracted from the aquifer, the ground water stock increases at the rate of recharge  $dG(t) = \tau R(t) + \omega \bar{a} = 0.3R(t) + 750,000$ .

- If  $R(t) = 34,711,500 - \frac{G(t)}{0.3}$ , the city should switch to **Case IV** at time  $t + 1$ .

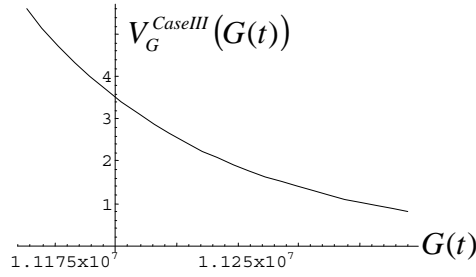
<sup>30</sup> See <http://mesonet.agron.iastate.edu/request/coop/fe.phtml> for more details.

- If  $34,711,500 - \frac{G(t)}{0.3} < R(t) < 84,535,600 - \frac{G(t)}{0.3}$ , the city should switch to **Case III** at time  $t + 1$ .
- If  $R(t) = 84,535,600 - \frac{G(t)}{0.3}$ , the city should switch to **Case II** at time  $t + 1$ .
- If  $R(t) > 84,535,600 - \frac{G(t)}{0.3}$ , the city should switch to **Case I** at time  $t + 1$ .
- Given that the current ground water stock must be  $G(t) < 11,163,400 = G^{CaseIV}$  in this case and that the maximum annual rainfall over the past 114 years is approximately 22,300,000 thousand gallons of water, it may be unlikely for the city to switch to **Case II** or **Case I** at time  $t + 1$  in this case.

**Case III:**  $g(t) > 0, b(t) = 0, a(t) = \bar{a}$

For **Case III** to be optimal, the current ground water stock must be such that  $G^{CaseIV} = 11,163,400 < G(t) < 26,110,700 = G^{CaseII}$ . To obtain the optimal rate of ground water extraction  $g^{CaseIII}(t)$ , one needs to solve (4.2.13) for the value function  $V^{CaseIII}(G(t))$ . This is done using NDSolve in Mathematica. However, two boundary conditions are required. Since (4.2.13) holds for  $G^{CaseIV} < G(t) < G^{CaseII}$ ,  $V_G^{CaseIII}(G^{CaseIV}) = C^b - C^g(G^{CaseIV})$  is used as a boundary condition. Another condition used is  $V^{CaseIII}(G^{CaseIV}) = V^{CaseIV}$ , where  $V^{CaseIV}$  is chosen so that  $V_G^{CaseIII}(G^{CaseII}) = \frac{C^a}{\omega}$  holds. In this case,  $V^{CaseIV} = 4,643,000,000$  is found.

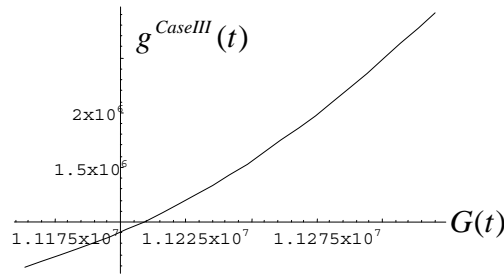
With (4.2.13),  $V_G^{CaseIII}(G^{CaseIV}) = 5.597$ , and  $V^{CaseIII}(G^{CaseIV}) = 4,643,000,000$ , the value function  $V^{CaseIII}(G(t))$  is obtained. The marginal user cost of ground water  $V_G^{CaseIII}(G(t))$  is then derived and shown in **Figure 18**. As can be seen, the marginal user cost is always above the unit cost of water that actually goes down to the aquifer by means of pumping in **Case III** ( $V_G^{CaseIII}(G(t)) > \frac{C^a}{\omega} = 0.8$ ), which is why the artificial recharge is done at full capacity.



**Figure 18.**  $V_G^{CaseIII}(G(t))$  for  $11,163,400 < G(t) < 26,110,700$

From (4.2.14), the optimal rate of ground water extraction  $g^{CaseIII}(t)$  is then obtained.

**Figure 19** shows the optimal ground water extraction  $g^{CaseIII}(t)$  that is increasing in the ground water stock  $11,163,400 < G(t) < 26,110,700$ .



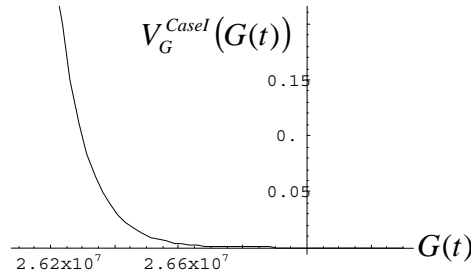
**Figure 19.**  $g^{CaseIII}(t)$  for  $11,163,400 < G(t) < 26,110,700$

- If  $R(t) = \left(\frac{g^{CaseIII}(t)-G(t)}{0.3}\right) + 84,535,600$ , the city should switch to **Case II** at time  $t + 1$ .
- If  $R(t) > \left(\frac{g^{CaseIII}(t)-G(t)}{0.3}\right) + 84,535,600$ , the city should switch to **Case I** at time  $t + 1$ .
- If  $R(t) = \left(\frac{g^{CaseIII}(t)-G(t)}{0.3}\right) + 34,711,500$ , the city should switch to **Case IV** at time  $t + 1$ .
- If  $R(t) < \left(\frac{g^{CaseIII}(t)-G(t)}{0.3}\right) + 34,711,500$ , the city should switch to **Case V** at time  $t + 1$ .
- Given that the minimum annual rainfall over the past 114 years is approximately 7,600,000 thousand gallons of water, since  $\max\left[\left(\frac{g^{CaseIII}(t)-G(t)}{0.3}\right) + 34,711,500\right]$  is found to be below 7,600,000, it may be unlikely for the city to switch to **Case IV** or **Case V** at time  $t + 1$  when the current ground water stock is  $11,163,400 < G(t) < 26,110,700$ .

**Case I:**  $g(t) > 0, b(t) = 0, a(t) = 0$

For **Case I** to be optimal, the current ground water stock must be such that  $G(t) > 26,110,700 = G^{CaseII}$ . To obtain the optimal rate of ground water extraction  $g^{CaseI}(t)$ , one needs to solve (4.2.5) for the value function  $V^{CaseI}(G(t))$ . Since (4.2.5) holds for  $G(t) > G^{CaseII}$ ,  $V_G^{CaseI}(G^{CaseII}) = \frac{c^a}{\omega}$  is used as a boundary condition. Another condition used is  $V^{CaseI}(G^{CaseII}) = V^{CaseIII}(G^{CaseII})$ , where  $V^{CaseIII}(G^{CaseII}) = 4,644,066,000$  is obtained from **Case III** above.

With (4.2.5),  $V_G^{CaseI}(G^{CaseII}) = 0.8$ , and  $V^{CaseI}(G^{CaseII}) = 4,644,066,000$ , the value function  $V^{CaseI}(G(t))$  is obtained. The marginal user cost of ground water  $V_G^{CaseI}(G(t))$  is then derived and shown in **Figure 20**:

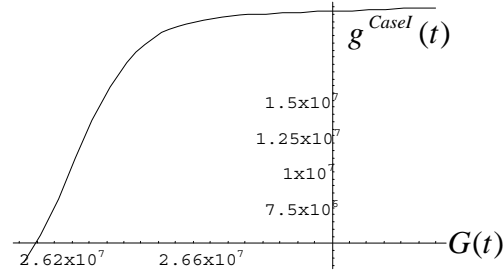


**Figure 20.**  $V_G^{CaseI}(G(t))$  for  $G(t) > 26,110,700$

As can be seen, the marginal user cost is always below the unit cost of water that actually goes down to the aquifer by means of pumping in **Case I** ( $V_G^{CaseI}(G(t)) < \frac{c^a}{\omega} = 0.8$ ), which is why no artificial recharge is done.

From (4.2.6), the optimal rate of ground water extraction  $g^{CaseI}(t)$  is then obtained. **Figure 21** shows the optimal ground water extraction  $g^{CaseI}(t)$  that is increasing in the ground water stock  $G(t) > 26,110,700$ .





**Figure 21.**  $g^{CaseI}(t)$  for  $G(t) > 26,110,700$

- If  $R(t) = \left(\frac{g^{CaseI}(t)-G(t)}{0.3}\right) + 87,035,600$ , the city should switch to **Case II** at time  $t+1$ .
- If  $\left(\frac{g^{CaseI}(t)-G(t)}{0.3}\right) + 37,211,500 < R(t) < \left(\frac{g^{CaseI}(t)-G(t)}{0.3}\right) + 84,535,600$ , the city should switch to **Case III** at time  $t+1$ .
- If  $R(t) = \left(\frac{g^{CaseI}(t)-G(t)}{0.3}\right) + 37,211,500$ , the city should switch to **Case IV** at time  $t+1$ .
- If  $R(t) < \left(\frac{g^{CaseI}(t)-G(t)}{0.3}\right) + 37,211,500$ , the city should switch to **Case V** at time  $t+1$ .
- Depending on the current ground water stock and the associated amount of ground water extracted at time  $t$ , **Case IV** and **Case V** may be unlikely at time  $t+1$ .

## 5 Conclusions

The paper examines separately the impacts of growing water demand and stochastic rainfall on intertemporal water use. The artificial ground water recharge role in alleviating water scarcity is investigated. In both the growing water demand model and the stochastic rainfall model of the intertemporal water use, the city should already be extracting from the less expensive water sources before extracting from the more expensive ones.

In the growing water demand model, because the water demand is assumed to grow over time, eventually ground water will not be able to completely satisfy the city's water needs. The city may have to resort to costly water from other cities to satisfy the excess demand. Since the artificial ground water recharge adds to the natural ground water, it can help prolong the period of not having to purchase costly water from other cities and reduce ground water extraction cost. However, only when the current ground water stock is

sufficiently small is the artificial ground water recharged used. This is due to the costs associated with pumping water underground.

In the stochastic rainfall model, it is not necessarily the case, though highly likely, that ground water is always extracted. Because rainfall is assumed to be stochastic, the city may find itself with significantly low levels of ground water stock and that it is relatively cheaper to temporarily rely on water from other cities while letting the ground water recharge. Water may be pumped to the aquifer to expedite the ground water recharge. When the current ground water stock is sufficiently large, ground water alone can satisfy the city's water needs regardless of current rainfall. Since the city can extract ground water without having to worry about current rainfall, no ground water conservation is needed in this case. Depending on how large the current ground water stock is, the artificial ground water recharge may not be done due to the costs associated with pumping water underground. Only when the city relies on both ground water and water from other cities at the same time should ground water conservation be practiced. The city should extract less ground water when current rainfall is small and extract more ground water otherwise. The ground water stock is kept constant so that the cost of using ground water equals the cost of obtaining water from other cities.

The growing water demand model and the stochastic rainfall model only apply to a growing city with small rainfall variability and a high-rainfall-variability city with small growth, respectively. This is because constant rainfall is assumed in the growing water demand model and constant water demand is assumed in the stochastic rainfall model. Since in the real world cities may be growing and at the same time experiencing highly variable rainfall, it would be ideal to look at the combined impact of growing water demand and stochastic rainfall on intertemporal water use. Solving model which accounts for both growing water demand and stochastic rainfall can prove challenging and may be included in future work.

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## CHAPTER 3. US FUEL ETHANOL DEMAND

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### Abstract

This paper tests the hypothesis that the derived demand for fuel ethanol in the US is perfectly elastic to see whether a long-run relationship exists between the ethanol price and the gasoline price. The Johansen and Juselius multivariate cointegration methodology finds no cointegration between the ethanol price and the gasoline price while the Gregory and Hansen residual-based tests for cointegration in models with regime shifts indicate that the long-run relationship between the ethanol price and the gasoline price exists with a possible structural break.

### 1 Introduction

“Alcohol and driving don’t mix. Or do they? Actually, they go together just fine, so long as your vehicle is the one consuming alcohol” (“Just the basics,” 2003). The history of ethanol as alternative transportation fuel in the US can be traced back to 1908 when Henry Ford came up with flexible fuel vehicle Model T that could run on ethanol, gasoline, or a combination of both. Adding ethanol raises the octane level and the oxygen level of gasoline, making the engines run smoothly and more cleanly without the need for lead, other additives such as methyl tertiary butyl ether (MTBE), or further gasoline refining. However, it was not until the 1970s when oil supply disruptions in the Middle East became a national threat and the US began to phase out lead in gasoline to protect public health that the interest in fuel ethanol started to rise. Tax benefits and incentive programs, such as a federal subsidy of \$0.54 per gallon for ethanol use<sup>31</sup> and varying supplemental state subsidies, were set up to

<sup>31</sup> This federal tax credit of \$0.54 per gallon for ethanol use was in effect until December 31, 2004. On January 1, 2005, a new federal tax credit of \$0.51 per gallon for ethanol use went into effect.

promote fuel ethanol (Rask, 1998). Nevertheless, MTBE was still the primary fuel additive used in the US at that time due to its better blending characteristics<sup>32</sup> when compared with ethanol and possibly the limit-pricing behavior of MTBE refiners as argued in Zhang, Vedenov, and Wetzstein (2007). Rask (1998) suggests that the high costs of transporting ethanol from the Midwest (production location) to other parts of the country might also be responsible for the lack of ethanol entry into the US fuel-additives market. The importance of proximity to ethanol production may also explain a finding by Gallagher, Otto, and Dikeman (2000) that ethanol blending is more profitable than MTBE blending in the Midwest markets provided that the tax credit is in effect.

The major switch to ethanol came in 2004 when the additive use of MTBE, made widespread by mandates of the 1990 Clean Air Act Amendments (CAAA) to reduce emissions in severely polluted regions, was banned<sup>33</sup> in the primary MTBE-using states California, New York, and Connecticut due to MTBE ground water contamination (“Status and impact,” 2003). By 2006, 19 states had partially or completely banned MTBE.<sup>34</sup> The US transition to fuel ethanol was further aided by the 2005 Energy Policy Act (EPAct). While eliminating the 1990 CAAA oxygenate requirement for reformulated gasoline in attempt to reduce the MTBE use, the 2005 EPAct established a national renewable fuel standard (RFS) mandating an increase in biofuel use from 4 billion gallons in 2006 to 7.5 billion gallons in 2012 (Neff, 2005). The RFS target was later modified by the 2007 Energy Independence and Security Act (EISAct) to increase from 9 billion gallons in 2008 to 36 billion gallons in 2022. Starting in 2016, the 2007 EISAct also requires that all of the increase in the RFS target be met entirely with cellulose-based biofuels setting a ceiling of 15 billion gallons of corn-based ethanol (Sissine, 2007). Though recent developments with cellulosic biomass conversion technologies allow ethanol to be produced from trees, grasses, and crop wastes which are abundantly present at a comparatively low cost,<sup>35</sup> these technologies are not yet profitable on a large scale due to their current poor conversion

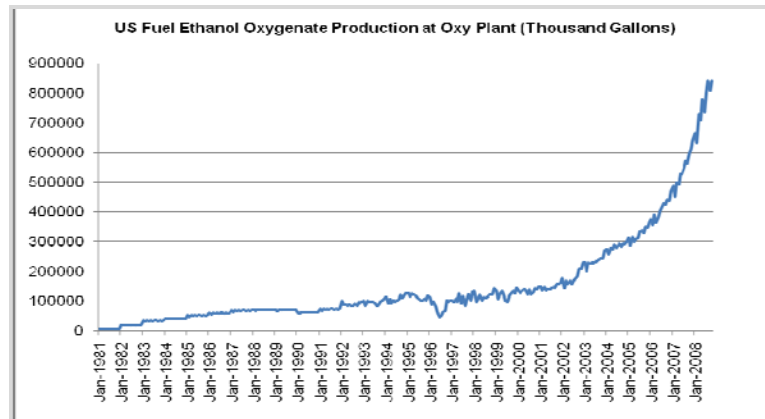
<sup>32</sup> See “MTBE fact sheet #3” (1998) for more details.

<sup>33</sup> See “State actions” (2004) for more details.

<sup>34</sup> See “State actions” (2004) for more details.

<sup>35</sup> Trees and grasses require less energy to grow than grains and do not have to be replanted every year (Biomass, 2006).

efficiency. As a result, there has yet to be a commercial cellulose-based ethanol plant in operation and ethanol is primarily produced from corn in the US. **Figure 22** shows US fuel ethanol production over time (Energy Information Administration).



**Figure 22. US fuel ethanol production (July 1981 – October 2008)**

Since ethanol is used in the US as an oxygenate, an octane enhancer, and a gasoline volume extender, the demand for fuel ethanol can be considered as being derived from both government regulations, which mandate oxygenate use, and the gasoline market. As MTBE was the oxygenate of choice for most blenders in satisfying the 1990 CAAA oxygenate requirements of a minimum 2.7 weight percent<sup>36</sup> oxygen in oxygenated gasoline and a minimum 2.1 weight percent<sup>37</sup> oxygen in reformulated gasoline, the regulatory demand for fuel ethanol prior to the MTBE bans may not be viewed as a lower level or a floor on fuel ethanol use. Only after the MTBE bans, when MTBE can no longer be used to satisfy the increasing RFS target, that the regulatory demand for fuel ethanol may be viewed as being perfectly inelastic at a quantity that is increasing over time. The data from the Energy Information Administration (EIA) shows that, in 2006, the US fuel ethanol consumption exceeded the 2005 EPAct RFS target of 4 billion gallons. In 2007, the 2005 EPAct RFS target of 7.5 billion gallons for 2012 was surpassed. Even under the 2007 EISAct RFS, the EIA, in its June 2007 Annual Energy Outlook, expects the US fuel ethanol consumption to

<sup>36</sup> This is equivalent to approximately 15 volume percent MTBE or 7.4 volume percent ethanol.

<sup>37</sup> This is equivalent to approximately 11.7 volume percent MTBE or 5.8 volume percent ethanol.

remain above the expanded and increasing RFS target until 2014. As a result, the regulatory demand for fuel ethanol is expected to have little impact on the ethanol price and may be ignored after the MTBE bans.

With E10 being the most widely used blend<sup>38</sup> in the US, the derived demand by blenders for fuel ethanol is somewhat restricted by the 10-percent-ethanol-90-percent-gasoline blending wall. If ethanol is viewed by blenders as being a perfect substitute for gasoline at roughly two-thirds the energy value and the ethanol price is roughly two-thirds the price of gasoline, the derived demand for fuel ethanol may be considered as being perfectly elastic at a price  $\overline{pe}$  that is related to the gasoline price  $pg$  for an ethanol quantity less than 10 percent of the finished fuel at the ethanol price  $\overline{pe}$  and the gasoline price  $pg$ . Blenders are indifferent between blending any amount of ethanol less than 10 percent of the finished fuel at the ethanol price  $\overline{pe}$  and the gasoline price  $pg$ . Other things being equal, blenders prefer ethanol to gasoline and would be willing to blend more of ethanol at an ethanol price  $pe < \overline{pe}$ . However, because of the 10-percent-ethanol-90-percent-gasoline blending wall, blenders can only increase the ethanol blend level to 10 percent of the finished fuel at the ethanol price  $pe < \overline{pe}$  and the gasoline price  $pg$ . As a result, the derived demand for fuel ethanol may be considered as being downward sloping for an ethanol price lower than  $\overline{pe}$ . **Figure 23** shows the derived demand curve for fuel ethanol where  $\overline{qe}$  is the ethanol quantity that is exactly equal to 10 percent of the finished fuel at the ethanol price  $\overline{pe}$  and the gasoline price  $pg$ . As long as ethanol is blended at a lower level than 10-percent-ethanol-90-percent-gasoline, the downward sloping portion of the derived demand may be ignored. A comparison between the US oxygenate plant production of fuel ethanol and the US finished

<sup>38</sup> Conventional gasoline engines are certified to operate on E10 without modification. Unlike flexible fuel vehicles that can operate on any ethanol-fuel mixture with ethanol concentrations of up to 85 percent (E85), conventional gasoline engines do not have a sensor to detect the ethanol-fuel ratio. So appropriate adjustments cannot be made to the engine's ignition timing and air-fuel mixture ratios to optimize performance and maintain emissions control if ethanol-fuel mixtures other than E10 are used.



motor gasoline product supplied<sup>39</sup> shows historical fuel ethanol use to be below the 10-percent-ethanol-90-percent-gasoline blending wall. The EIA, in its 2007 Annual Energy

Outlook, projects fuel ethanol use to account for approximately 8 percent of the total fuel use in 2030. As a result, the derived demand for fuel ethanol may be viewed simply as a relationship between the ethanol price and the gasoline price. **Figure 24** shows this possible relationship between the ethanol price and the gasoline price (Oxy Fuel News, Ethanol & Biodiesel News, and Energy Information Administration).

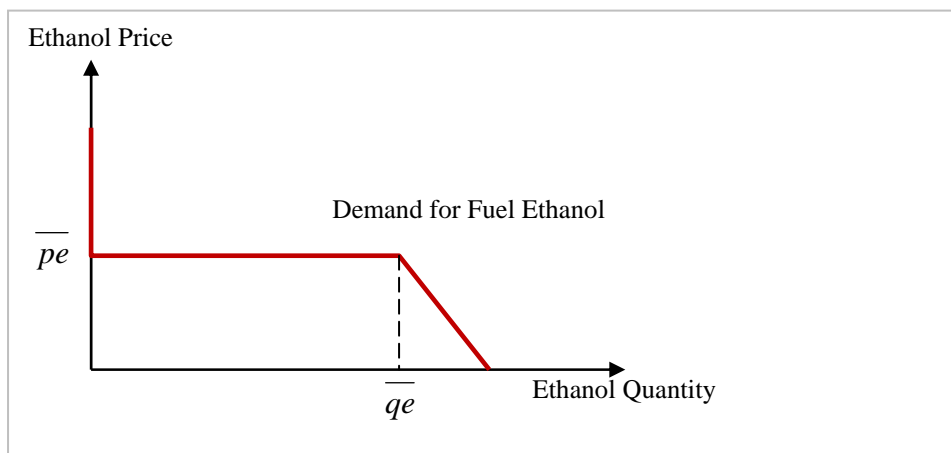


Figure 23. The derived demand for fuel ethanol when ethanol is viewed as being a perfect substitute for gasoline

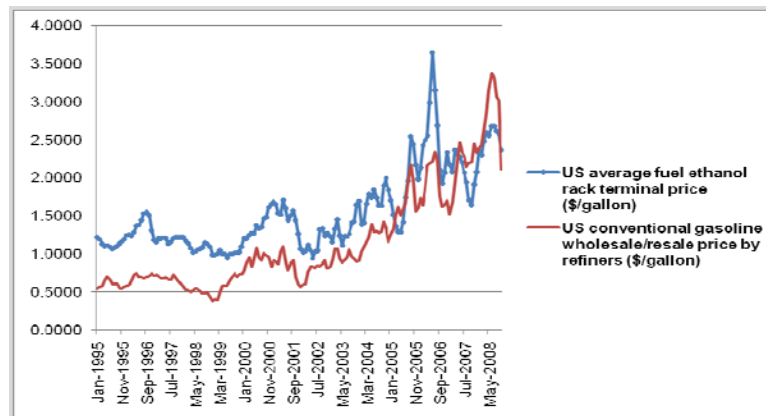


Figure 24. US fuel ethanol price and US conventional gasoline price (January 1995 – October 2008)

<sup>39</sup> Both statistics are obtained from the EIA.

This paper tests the hypothesis that the derived demand for fuel ethanol is perfectly elastic to see whether a long-run relationship exists between the ethanol price and the gasoline price; and in examining how the demand may have been affected by some of the 2005 EPA mandates and the significantly higher gasoline price in recent years, which may have caused a structural break in the long-run relationship between the ethanol price and the gasoline price. The Johansen and Juselius multivariate cointegration methodology is employed in estimating the demand, i.e. searching for the long-run relationship between the ethanol price and the gasoline price, while the Gregory and Hansen (1996) tests for cointegration in models with regime shifts are utilized to see if a long-run equilibrium ethanol price equation exists with a structural break. The model is applied to an empirical analysis of the market between January 1995 and October 2008, the period of which the ethanol price data and the gasoline price data are available for the US.

Higgins, Bryant, Outlaw, and Richardson (2006), as part of their study of US fuel ethanol pricing, also examine the relationship between the ethanol price and the gasoline price using the time series techniques of cointegration. However, the analysis is for the period of June 1989 to August 2005. Based upon the obtained long-run relationship, ethanol and gasoline are found to be weak substitutes. Higgins et al. explain this weak substitutability as a likely result of the competing complementarity between ethanol and gasoline. Serra, Zilberman, Gil, and Goodwin (2008) use daily futures prices for corn, ethanol, and crude oil observed from July 21, 2005 to May 15, 2007 in characterizing the corn-ethanol-oil price relationships. To allow for nonlinearities in the process of price adjustment towards long-run relationships, smooth transition vector error correction model (STVECM) is used. Though a long-run relationship is found between the ethanol, corn, and crude oil prices, not much attention is paid to the cointegrating vector itself.

Studies focusing on other aspects of the time series properties of US fuel ethanol pricing, besides cointegration, are for example the followings. Zhang and Wetzstein (2008), in addressing the food-versus-fuel issues, examine the relationships between the weekly price series for US ethanol, corn, conventional gasoline, and oil from the last week of March 1989 through the first week of December 2007. A vector autoregression (VAR) model is used to estimate the evolution of the price series, while a multivariate generalized autoregressive

conditional heteroskedasticity (MGARCH) model is used to estimate the conditional volatilities of the log price changes. Zhang, Vedenov, and Wetzstein (2007) develop a structural vector autoregression (SVAR) model of the US fuel ethanol market to evaluate the validity of the limit-price hypothesis on the part of MTBE refiners as an explanation of the lack of ethanol entry into the US fuel-additives market before MTBE bans. Other studies of the US fuel ethanol market that may be of some interest include Elobeid and Tokgoz (2006) which uses a multi-market international ethanol model calibrated on 2005 market data and policies to study the impact of the US trade liberalization and removal of federal ethanol tax credit on the US and Brazilian ethanol markets.

This paper is organized as follows. The next section presents econometric methodology and specifies model used in finding a long-run relationship between the ethanol price and the gasoline price. Section 3 checks whether the time series are integrated of the same order as required by the concept of cointegration. Section 4 reports results from testing the null hypothesis of no cointegration. Section 5 tests if a cointegrating relationship between the ethanol price and the gasoline price exists with a structural break. Section 6 concludes.

## **2 Econometric Methodology and Model Specification**

Since this paper explores relationships among time series variables, it is necessary to first check if the time series variables are stationary. Spurious relationships may be encountered when running a regression involving two or more non-stationary time series variables. However, if the non-stationary time series variables are integrated of the same order, it is possible that their linear combination is stationary. When their linear combination is stationary, the non-stationary time series variables of interest are said to be cointegrated (Engle & Granger, 1980). Unit root tests are employed in testing whether the time series variables are stationary and ensuring that they have the same order of integration in case they are non-stationary.

After checking that the time series variables have the same order of integration, the next step is to check whether they are cointegrated. This is done using the Johansen and Jesulius multivariate cointegration methodology. Unlike the Engle and Granger residual-

based test for cointegration, the Johansen and Juselius methodology allows for possibility of having more than one cointegrating vector among the time series variables of interest and for inferences to be made on the parameters.<sup>40</sup> It is the preferred methodology reported by Higgins et al. (2006) in finding cointegrating relationships.

Let  $Y_t$  be a  $K \times 1$  vector of the time series variables of interest that are integrated of the same order  $I(1)$ . A vector error correction model (VECM) of the form

$$\Delta Y_t = \alpha(\beta'Y_{t-1} + \mu) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \quad (1)$$

is employed so that the Johansen and Juselius methodology can be carried out.  $\beta$  is a  $K \times r$  matrix of parameters corresponding to the cointegrating relationships among the time series variables  $Y_t$ .  $\mu$  is an  $r \times 1$  vector of constants that are also part of the cointegrating relationships.  $\alpha$  is a  $K \times r$  matrix of parameters corresponding to the short-run adjustments to the time series variables  $Y_t$  given a departure from the long-run relationships.  $\Gamma_1, \dots, \Gamma_{p-1}$  are  $K \times K$  matrices of parameters.  $\varepsilon_t$  is a  $K \times 1$  vector of normally distributed errors that are serially uncorrelated but have contemporaneous covariance matrix  $\Omega$ .  $p$  is chosen based on the information criteria obtained from running a VAR model that underlies the above VECM (1). It is the number of lags used in the underlying VAR.

Engle and Granger (1987) shows that if the time series variables  $Y_t$  are cointegrated, the matrices  $\beta$  and  $\alpha$  in (1) have rank  $0 < r < K$  where  $r$  is the number of linearly independent cointegrating vectors. If, on the other hand, the time series variables  $Y_t$  are not cointegrated, the cointegration rank  $r$  equals zero. The trace statistic, based on the Johansen and Juselius maximum likelihood estimator of the parameters of a cointegrating VECM (1), is used to determine the cointegration rank  $r$ .

Given the number of lags  $p$  and the cointegration rank  $r$ , (1) is fitted using maximum likelihood methods. The log-likelihood function for (1) can be maximized more easily by concentrating it in the following form

<sup>40</sup> The Engle and Granger approach results in inefficient estimation of the existing cointegrating relationship (Campiche, Bryant, Richardson, & Outlaw, 2006).

$$L = -\frac{1}{2} \left\{ TK \ln(2\pi) + T \ln(|\Omega|) + \sum_{t=1}^T (Z_{0t} - \alpha \tilde{\beta}' Z_{1t} - \psi Z_{2t})' \Omega^{-1} (Z_{0t} - \alpha \tilde{\beta}' Z_{1t} - \psi Z_{2t}) \right\} \quad (2)$$

where  $T$  is the sample size;  $Z_{0t} = \Delta Y_t$  is  $K \times 1$ ;  $Z_{1t} = (Y'_{t-1}, 1)'$  is  $(K+1) \times 1$ ;

$Z_{2t} = (\Delta Y'_{t-1}, \dots, \Delta Y'_{t-p+1})'$  is  $K(p-1) \times 1$ ;  $\psi = (\Gamma_1, \dots, \Gamma_{p-1})$  is  $K \times K(p-1)$ ; and  $\tilde{\beta} = (\beta', \mu)'$  is  $(K+1) \times r$ . Johansen (1995) shows how  $\psi$  can be expressed analytically in terms of  $\alpha$ ,  $\beta$ , and the data so that (2) is concentrated further as follows

$$L = -\frac{1}{2} \left\{ TK \ln(2\pi) + T \ln(|\Omega|) + \sum_{t=1}^T (R_{0t} - \alpha \tilde{\beta}' R_{1t})' \Omega^{-1} (R_{0t} - \alpha \tilde{\beta}' R_{1t}) \right\} \quad (3)$$

where  $M_{ij} = T^{-1} \sum_{t=1}^T Z_{it} Z'_{jt}$ ,  $i, j \in \{0, 1, 2\}$ ;  $R_{0t} = Z_{0t} - M_{02} M_{22}^{-1} Z_{2t}$  is the residuals obtained

from the regression of  $Z_{0t}$  on  $Z_{2t}$ ; and  $R_{1t} = Z_{1t} - M_{12} M_{22}^{-1} Z_{2t}$  is the residuals obtained

from the regression of  $Z_{1t}$  on  $Z_{2t}$ . Although the estimate of  $\alpha \tilde{\beta}'$  is obtained from (3), not all the parameters in  $\alpha$  and  $\tilde{\beta}$  are identified. This is because the product of  $\alpha Q$  and

$Q^{-1} \tilde{\beta}'$ , where  $Q$  is a nonsingular  $r \times r$  matrix, produces the same value  $\alpha \tilde{\beta}'$ . So

substituting  $\tilde{\alpha} = \alpha Q$  and  $\tilde{\beta}' = Q^{-1} \tilde{\beta}'$  into (3) for  $\alpha$  and  $\tilde{\beta}$  would not change the value of the log likelihood. To identify  $\alpha$  and  $\tilde{\beta}$ , some a priori identification restrictions are required. Johansen (1995) proposes a normalization method that places  $r^2$  linearly independent restrictions

$$\tilde{\beta}' = (I_r, \tilde{\beta}') \quad (4)$$

, where  $I_r$  is the  $r \times r$  identity matrix and  $\tilde{\beta}$  is a  $(K-r) \times r$  matrix of identified parameters,

on the parameters in  $\tilde{\beta}$  and shows how these estimates of the identified parameters in  $\tilde{\beta}$

converge at a faster rate than the estimates of the short-run parameters in  $\alpha$  and  $\Gamma_i$ , allowing

the distribution of the estimates of the short-run parameters in  $\alpha$  and  $\Gamma_i$  to be derived

conditional on the estimated  $\tilde{\beta}$ .

For a given value of  $\tilde{\beta}$ , the regression of  $R_{0t}$  on  $\tilde{\beta}'R_{1t}$  then yields

$$\alpha(\tilde{\beta}) = S_{01}\tilde{\beta}(\tilde{\beta}'S_{11}\tilde{\beta})^{-1} \quad (5)$$

$$\Omega(\tilde{\beta}) = S_{00} - S_{01}\tilde{\beta}(\tilde{\beta}'S_{11}\tilde{\beta})^{-1}\tilde{\beta}'S_{10}$$

$$S_{ij} = (1/T) \sum_{t=1}^T R_{it}R'_{jt}, \quad i, j \in \{0,1\}$$

Using the solutions (5) in (3), Johansen (1995) shows that the estimates  $\hat{\tilde{\beta}}$  of the parameters in  $\tilde{\beta}$  are given by the  $r$  eigenvectors  $v_1, \dots, v_r$  corresponding to the  $r$  largest eigenvalues  $\lambda_1, \dots, \lambda_r$  that solve the generalized eigenvalue problem

$$\left| \lambda_i S_{11} - S_{10} S_{00}^{-1} S_{01} \right| = 0 \quad (6)$$

To identify these eigenvectors  $v_1, \dots, v_r$ , (4) is imposed. The eigenvectors  $v_1, \dots, v_r$  are normalized such that

$$\lambda_i S_{11} v_i = S_{10} S_{00}^{-1} S_{01} v_i \quad (7)$$

$$v'_i S_{11} v_j = \begin{cases} 1 & , i = j \\ 0 & , otherwise \end{cases}$$

Let  $\hat{\lambda}_i$  be the eigenvalues that solve (6) and (7), the log-likelihood function (3) at the optimum is given by

$$L^* = -\frac{1}{2}T \left\{ K \ln(2\pi) + K + \ln(|S_{00}|) + \sum_{i=1}^r \ln(1 - \hat{\lambda}_i) \right\} \quad (8)$$

The asymptotic distribution of  $\hat{\tilde{\beta}}$  is shown to be mixed Gaussian and the variance-covariance (VCE) matrix of  $\hat{\tilde{\beta}}$  can be consistently estimated by

$$\left( \frac{1}{T-d} \right) (I_r \otimes H_J) \left\{ (\hat{\alpha}' \hat{\Omega}^{-1} \hat{\alpha}) \otimes (H'_J S_{11} H_J) \right\}^{-1} (I_r \otimes H_J)' \quad (9)$$

where  $H_J = (0'_{r \times (K+1-r)}, I_{K+1-r})'$  is  $(K+1) \times (K+1-r)$ ;  $\hat{\alpha} = S_{01} \hat{\tilde{\beta}} (\hat{\tilde{\beta}}' S_{11} \hat{\tilde{\beta}})^{-1}$  is the estimates of the parameters in  $\alpha$  conditional on  $\hat{\tilde{\beta}}$ ;  $\hat{\Omega} = S_{00} - \hat{\alpha} \hat{\tilde{\beta}}' S_{10}$  is the estimate of  $\Omega$  conditional on  $\hat{\tilde{\beta}}$ ;  $d$  is the degrees of freedom of the model calculated as the integer part of

$(n_{parms} - r^2) / K$ ; and  $n_{parms} = Kr + (K + 1)r + K\{K(p - 1)\}$  is the total number of parameters in (1). The estimated VCE matrix of  $\hat{\alpha}$  is given by

$$\left(\frac{1}{T-d}\right)\hat{\Omega} \otimes \hat{\Sigma}_B \quad (10)$$

where  $\hat{\Sigma}_B = (\hat{\beta}'S_{11}\hat{\beta})^{-1}$  (Stata Time-series, 2007, pp. 398).

Since this paper examines whether a long-run relationship exists between the ethanol price and the gasoline price, the time series variables of interest are  $Y_t = (pe_t \quad pg_t)'$ .  $pe_t$  is the US average fuel ethanol rack terminal price in dollars per gallon at time  $t$  obtained from Oxy Fuel News<sup>41</sup> and Ethanol & Biodiesel News.<sup>42</sup>  $pg_t$  is the US conventional gasoline wholesale/resale price by refiners in dollars per gallon at time  $t$  obtained from the EIA.<sup>43</sup> Both prices are observed from January 1995 to October 2008 (see **Figure 24**).

### 3 Stationarity and Integration Properties of the Data

The augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test are used in testing for unit roots in the autoregressive representation of each individual time series variable  $y_t$ . The ADF test is carried out in the following context

$$y_t = \zeta + \delta t + \rho y_{t-1} + \gamma_1 \Delta y_{t-1} + \dots + \gamma_m \Delta y_{t-m} + u_t \quad (11)$$

where  $u_t$  is an independently and identically distributed zero-mean error terms. The lagged values of the difference of the time series variable  $\Delta y_{t-1}, \dots, \Delta y_{t-m}$  are included in (11) to accommodate any serial correlation in the disturbances. The lag length  $m$  is chosen to minimize the following information criteria

$$IC(m) = \ln\left(\frac{e'e}{T - m_{\max} - K^*}\right) + (m + K^*)\left(\frac{A^*}{T - m_{\max} - K^*}\right) \quad (12)$$

where  $e$  is the residuals obtained from (11);  $m_{\max}$  is the largest lag length being considered and equals the integer part of  $[12\left(\frac{T}{100}\right)^{0.25}]$ ;  $K^*$  equals one for random walk, two for random walk with drift, and three for trend stationary; and  $A^*$  equals two for Akaike information criterion (AIC) and  $\ln(T - m_{\max} - K^*)$  for Schwartz Bayesian information criterion (SBIC).

<sup>41</sup> See <http://proquest.umi.com/pqdweb?RQT=318&pmid=32874&cfc=1> for more details.

<sup>42</sup> See <http://proquest.umi.com/pqdweb?RQT=318&pmid=68404> for more details.

The null hypothesis is that the time series variable  $y_t$  has a unit root or, in the context of (11),  $\rho$  equals one. The alternative hypothesis is that  $y_t$  is stationary. If the test statistic  $ADF_{t-stat} = \frac{\hat{\rho}-1}{Est.Std.Error(\hat{\rho})}$  is less than the critical values, the null hypothesis is rejected and  $y_t$  is stationary.

The PP test is carried out in the following context

$$y_t = \zeta + \delta t + \rho y_{t-1} + u_t \quad (13)$$

Unlike the ADF test, any serial correlation in the disturbances is ignored in (13) but is

accounted for in the test statistics  $Z_{t-stat} = \sqrt{\frac{c_0}{a}} \left( \frac{\hat{\rho}-1}{v} \right) - \frac{1}{2} (a - c_0) \frac{Tv}{\sqrt{as^2}}$  and

$Z_{\rho-stat} = T(\hat{\rho}-1) - \frac{1}{2} \left( \frac{T^2 v^2}{s^2} \right) (a - c_0)$  where  $T$  is the number of observations;  $v^2$  is the estimated asymptotic variance of  $\hat{\rho}$ ;  $c_0 = [(T - K^*)/T] s^2$ ;  $K^*$  is the number of regressors;

$s^2 = \frac{\sum_{t=1}^T e_t^2}{T-K^*}$ ;  $e_t$  is the residuals obtained from (14);  $a = c_0 + 2 \sum_{j=1}^{L^*} \left( 1 - \frac{j}{L^*+1} \right) c_j$ ;  $L^*$  equals the

integer part of  $[4 \left( \frac{T}{100} \right)^{\frac{2}{9}}]$ ; and  $c_j = \frac{1}{T} \sum_{s=j+1}^T e_t e_{t-s}$ ,  $j = 0, \dots, m$  is the  $j^{th}$  autocovariance of the

residuals  $e_t$ . If these test statistics are less than the critical values, the null hypothesis of  $y_t$  having a unit root (i.e.  $H_0 : \rho = 1$ ) is rejected and  $y_t$  is stationary.

Both the ADF test (with the lag length chosen based on AIC) and the PP test indicate that, at 1% and 5% significance levels, the ethanol price  $pe_t$  and the gasoline price  $pg_t$  are all non-stationary and can be made stationary by taking the first difference. Therefore, they are integrated of the same order  $I(1)$ . Error! Reference source not found. reports the results of the ADF test (i.e. note that the autoregressive representations of  $pe_t$  and of  $pg_t$  include a trend), while **Table 4** reports the results of the PP test (i.e. note that the autoregressive representations of  $pe_t$  and of  $pg_t$  include a trend). All test results are obtained from Stata 10 output. Significant values at 1% and 5% levels are denoted by \* and \*\* respectively. The

<sup>43</sup> See <http://tonto.eia.doe.gov/dnav/pet/hist/a163700002m.htm> for more details.



autoregressive representation of  $pe_t$  and the autoregressive representation of  $pg_t$  include a trend.

**Table 3. ADF results**

| time series    | m chosen based on AIC  |                       |                   |                   |
|----------------|------------------------|-----------------------|-------------------|-------------------|
|                | m                      | $ADF_{t-stat}$        | 1% critical value | 5% critical value |
| $pe_t$ (trend) | 2                      | -3.404                | -4.019            | -3.442            |
| $pg_t$ (trend) | 10                     | -1.121                | -4.022            | -3.443            |
| $\Delta pe_t$  | 1                      | -9.543 <sup>***</sup> | -2.592            | -1.950            |
| $\Delta pg_t$  | 9                      | -4.380 <sup>***</sup> | -2.593            | -1.950            |
| time series    | m chosen based on SBIC |                       |                   |                   |
|                | m                      | $ADF_{t-stat}$        | 1% critical value | 5% critical value |
| $pe_t$ (trend) | 1                      | -4.289 <sup>***</sup> | -4.019            | -3.441            |
| $pg_t$ (trend) | 1                      | -3.470 <sup>**</sup>  | -4.019            | -3.441            |
| $\Delta pe_t$  | 1                      | -9.543 <sup>***</sup> | -2.592            | -1.950            |
| $\Delta pg_t$  | 0                      | -7.809 <sup>***</sup> | -2.591            | -1.950            |

**Table 4. PP results**

| time series    | $Z_{t-stat}$          | 1% critical | 5% critical | $Z_{\rho-stat}$         | 1% critical | 5% critical |
|----------------|-----------------------|-------------|-------------|-------------------------|-------------|-------------|
|                |                       | Value       | value       |                         | value       | value       |
| $pe_t$ (trend) | -3.312                | -4.018      | -3.441      | -20.837                 | -27.833     | -20.960     |
| $pg_t$ (trend) | -2.869                | -4.018      | -3.441      | -15.529                 | -27.833     | -20.960     |
| $\Delta pe_t$  | -8.795 <sup>***</sup> | -2.591      | -1.950      | -92.916 <sup>***</sup>  | -13.428     | -7.943      |
| $\Delta pg_t$  | -7.595 <sup>***</sup> | -2.591      | -1.950      | -107.154 <sup>***</sup> | -13.428     | -7.943      |

#### 4 Cointegration Analysis

Since the ethanol price  $pe_t$  and the gasoline price  $pg_t$  are integrated of the same order  $I(1)$ , the Johansen and Juselius multivariate cointegration methodology is employed to check if a long-run relationship exists between them. Firstly, *varsoc* command in Stata 10 is used to select the lag length  $p$  in the VECM (1) where  $Y_t = (pe_t \quad pg_t)'$ . The lag-order

selection statistics obtained from *varsoc* output for a series of VAR models of order 1, ..., 13 that underlie the VECM (1) where  $Y_t = (pe_t \quad pg_t)'$  are reported in **Table 5** (i.e. note that the largest lag length of 13 being considered equals the integer part of  $[12(\frac{T}{100})^{0.25}]$  where the number of observations  $T$  equals 166 in this case). Values indicating the optimal lag are in bold.

**Table 5. Lag-order selection statistics**

| varsoc pe pg, maxlag(13) noconstant |         |              |    |         |                |                     |                 |                 |
|-------------------------------------|---------|--------------|----|---------|----------------|---------------------|-----------------|-----------------|
| Selection order criteria            |         |              |    |         |                |                     |                 |                 |
| Sample:                             | 1996m2  |              |    | 2008m10 |                | Number of obs = 153 |                 |                 |
| Lag                                 | LL      | LR           | Df | P       | FPE            | AIC                 | HQIC            | SBIC            |
| 1                                   | 164.358 | .            | 4  | .       | .000421        | -2.09619            | -2.06401        | -2.01696        |
| 2                                   | 178.902 | 29.087       | 4  | 0.000   | .000367        | -2.23402            | -2.16965        | <b>-2.07556</b> |
| 3                                   | 185.593 | 13.383       | 4  | 0.010   | .000354        | -2.2692             | <b>-2.17265</b> | -2.03151        |
| 4                                   | 188.19  | 5.1925       | 4  | 0.268   | .000361        | -2.25085            | -2.12211        | -1.93394        |
| 5                                   | 188.785 | 1.19         | 4  | 0.880   | .000378        | -2.20634            | -2.04542        | -1.8102         |
| 6                                   | 196.406 | 15.243       | 4  | 0.004   | .00036         | -2.25367            | -2.06057        | -1.77831        |
| 7                                   | 199.017 | 5.2218       | 4  | 0.265   | .000367        | -2.23552            | -2.01023        | -1.68093        |
| 8                                   | 205.422 | 12.809       | 4  | 0.012   | .000356        | -2.26695            | -2.00948        | -1.63313        |
| 9                                   | 211.717 | 12.591       | 4  | 0.013   | .000345        | -2.29696            | -2.00731        | -1.58391        |
| 10                                  | 221.653 | 19.872       | 4  | 0.001   | .00032         | -2.37455            | -2.05271        | -1.58228        |
| 11                                  | 226.89  | 10.474       | 4  | 0.033   | .000315        | -2.39072            | -2.0367         | -1.51922        |
| 12                                  | 232.974 | 12.169       | 4  | 0.016   | .000307        | -2.41796            | -2.03176        | -1.46724        |
| 13                                  | 243.119 | <b>20.29</b> | 4  | 0.000   | <b>.000284</b> | <b>-2.49829</b>     | -2.0799         | -1.46834        |
| Endogenous:                         | pe pg   |              |    |         |                |                     |                 |                 |
| Exogenous:                          | -       |              |    |         |                |                     |                 |                 |

As can be seen, the Schwartz Bayesian information criterion (SBIC) chooses two lags. The Hannan-Quinn information criterion (HQIC) chooses three lags. The sequential likelihood-ratio (LR) test chooses thirteen lags. Since the data are monthly, thirteen lags<sup>44</sup> are selected in this case.

<sup>44</sup> Although cointegration between the ethanol price and the gasoline price is found to exist with a structural break, the Johansen and Juselius maximum likelihood estimator of the VECM (1) with the cointegrating vector specified as in (14) produces the estimates of the structural change parameters  $\beta_3$  and  $\mu_2$  that are not significantly different from zero when the VECM (1) is specified with three lags (i.e.  $p = 3$ ). When VECM (1) is specified with two lags (i.e.  $p = 2$ ), it is not certain if cointegration between the ethanol price and the gasoline price exists with a structural break even though the Gregory and Hansen (1996) tests for cointegration in models with regime shifts fail to reject the null hypothesis of no cointegration. This is because cointegration is also found in the absence of a structural break.

Secondly, *vecrank* command in Stata 10 is used to determine the cointegration rank  $r$  in the VECM (1) where  $Y_t = (pe_t \quad pg_t)'$ . The trace statistic method implemented in *vecrank* is based on the eigenvalues  $\hat{\lambda}_1, \dots, \hat{\lambda}_K$  used in computing the optimum log-likelihood function (8) and sorted from the largest  $\hat{\lambda}_1$  to the smallest  $\hat{\lambda}_K$ . The null hypothesis of the trace statistic method is that there are no more than  $r$  cointegrating equations in the system. Restricting the cointegration rank to be  $r$  or less implies that the remaining eigenvalues  $\hat{\lambda}_{r+1}, \dots, \hat{\lambda}_K$  are zero. As a result, large values of the trace statistic  $-T \sum_{i=r+1}^K \ln(1 - \hat{\lambda}_i)$  are evidence against the null hypothesis that there are  $r$  or fewer cointegrating equations in the system. The trace statistic method starts testing at  $r = 0$  and accepts as an estimator  $\hat{r}$  of the true number of cointegrating equations the first value of  $r$  for which the null hypothesis cannot be rejected. Error! Reference source not found. reports *vecrank* output for the VECM (1) where  $Y_t = (pe_t \quad pg_t)'$  and the lag length  $p$  equals 13.

**Table 6. Trace statistics used in determining the cointegration rank  $r$**

| vecrank pe pg, trend(rconstant) lags(13) |       |           |            |                 |                   |                     |
|--|-------|-----------|------------|-----------------|-------------------|---------------------|
| Johansen tests for cointegration         |       |           |            |                 |                   |                     |
| Trend: rconstant                         |       |           |            |                 |                   |                     |
| Sample: 1996m2 2008m10                   |       |           |            |                 |                   |                     |
| maximum rank                             | Parms | LL        | eigenvalue | trace statistic | 5% critical value |                     |
| 0  | 48    | 239.18949 | .          | <b>16.2848</b>  | 19.96             | Number of obs = 153 |
| 1  | 52    | 245.55667 | 0.07986    | 3.5504          | 9.42              | lags = 13           |
| 2  | 54    | 247.33189 | 0.02294    |                 |                   |                     |

As the trace statistic of 16.28 is less than the 5% critical value of 19.96, the null hypothesis of no cointegration (i.e. cointegration rank being zero) cannot be rejected. However, the failure to reject the null hypothesis of no cointegration does not necessarily imply the lack of long-run relationship between the ethanol price  $pe_t$  and the gasoline price  $pg_t$ . The ethanol price and the gasoline price may be cointegrated in the sense that their linear combination (i.e. the cointegrating vector) is stationary but has shifted at one point in time (Gregory & Hansen, 1996). In fact, the rapid growth enjoyed by the US fuel ethanol industry due to the

significantly higher gasoline prices and changes to the industry in recent years may lead one to suspect this possibility.

## 5 Cointegration with Structural Break

To check if a long-run relationship between the ethanol price and the gasoline price exists with a structural break, the Gregory and Hansen (1996) residual-based tests for cointegration in models with regime shifts are employed. These tests are designed to test the null hypothesis of no cointegration against the alternative hypothesis of cointegration in the presence of a possible regime shift and do not require information regarding the timing of a break. Structural change is modeled using a dummy variable. Following Gregory and Hansen's procedure, a long-run equilibrium ethanol price equation which allows for a possible regime shift is shown below

$$pe_t = \beta_{2,\tau} pg_t + \beta_{3,\tau} (sbd_{\tau,t} * pg_t) + \mu_{1,\tau} + \mu_{2,\tau} sbd_{\tau,t} + v_{\tau,t} \quad (14)$$

where  $sbd_{\tau,t}$  is the structural break dummy that equals one for  $t \geq \tau$  and zero otherwise.

(14) is estimated by ordinary least squares (OLS) for each break point  $\tau \in [jan95, oct08]$  yielding the residuals  $\hat{v}_{\tau,t}$ . The  $ADF_{\tau}$  test statistic associated with each break point  $\tau$  is obtained from the autoregressive representation<sup>45</sup> (11) of these residuals  $\hat{v}_{\tau,t}$ . The Gregory and Hansen (GH) test statistic is computed as  $GH = \inf_{\tau \in [jan95, oct08]} ADF_{\tau}$  which, in this case, equals  $ADF_{may07} = -5.52$ . Since the GH test statistic of -5.52 is smaller than the 1%, 5%, and 10% critical values<sup>46</sup> of -5.47, -4.95, and -4.68, the null hypothesis of no cointegration is rejected. Also because no cointegration is found in section 5, it can be concluded that the long-run relationship between the ethanol price and the gasoline price exists with a possible structural break at the estimated break point May 2007. As can be seen in **Figure 24**, the gasoline price was below the ethanol price in the pre-May 2007 period and was above the ethanol price in the post-May 2007 period.

Looking back at that time period, one can identify two potential shocks that may have

<sup>45</sup> Lag length is chosen to minimize AIC. Trend is included.

<sup>46</sup> See Table 1 in Gregory and Hansen (1996).

caused this break in the cointegrating relationship: the 2005 EPAct and the crude oil price increase beginning in 2005. The 2005 EPAct was passed and signed into law in late July 2005. Though not directly banning MTBE, it provided no protection for liability related to MTBE use, allowing MTBE liability suits to be moved to federal court. Some blenders, in fears of these potential legal liabilities, may have decided to limit or stop MTBE use.<sup>47</sup> Since the 2005 EPAct also eliminated the oxygenate requirement of a minimum of 2.1 weight percent oxygen in reformulated gasoline (i.e. a key defense against the liability suits for MTBE blenders), ethanol remains the main surviving fuel additive for increasing octane. However, because ethanol and MTBE have different physical and chemical properties, the substitution of ethanol for MTBE may have not replaced all of the gasoline volume lost by removing MTBE.<sup>48</sup> Besides, because most MTBE was used on the East and West Coasts while ethanol has been largely produced in the Midwest, the substitution of ethanol for MTBE may have occurred slowly in these regions. Unlike MTBE, ethanol cannot be blended at the refinery and distributed with gasoline through pipelines due to water absorption and materials incompatibilities (“MTBE fact sheet #3,” 1998). If ethanol-blended gasoline is exposed to water, phase separation will occur. In addition, ethanol can damage pipeline seals and even induce cracking in pipeline steel (Farrell et al., 2007). Morrow, Griffin, and Matthews (2006) estimate the combined cost of transporting ethanol from production plants to fueling stations to be 10-13 cents per gallon over the cost of transporting petroleum fuels. According to the EIA, the West Texas Intermediate (WTI) crude oil price<sup>49</sup> fluctuated from about \$63 per barrel in May 2007 to a record high of \$134 per barrel in June 2008 due to the increase in global oil demand and the disruptions to oil supply. The resulting significantly higher gasoline price may have caused blenders to increase their use of ethanol as a gasoline volume extender. **Figure 25** shows blenders’ decreasing use of MTBE and increasing use of ethanol over time (Energy Information Administration).

<sup>47</sup> According to the National Petrochemical and Refiners Association (NPRA), MTBE use in reformulated gasoline accounted for as much as 11 percent of the reformulated gasoline supply at its peak.

<sup>48</sup> The NPRA indicates ethanol’s properties generally cause ethanol to replace only about 50 percent of the gasoline volume lost when MTBE is removed.

<sup>49</sup> See <http://tonto.eia.doe.gov/dnav/pet/hist/rwtcm.htm> for more details.

Now that the long-run relationship between the ethanol price and the gasoline price has been shown to exist with a possible structural break at the estimated break point May 2007, the VECM (1) where  $Y_t = (pe_t \quad pg_t)'$ , the number of lags  $p = 13$ , and the cointegration rank  $r = 1$  can be re-estimated with the presence of a structural break in the cointegrating equation. The new model is

$$\Delta Y_t = \alpha \left\{ \beta' Y_{t-1} + \beta_3 (sbd_{may07,t} * pg_t) + \mu_1 + \mu_2 sbd_{may07,t} \right\} + \sum_{i=1}^{12} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \quad (15)$$

where  $Y_t = (pe_t \quad pg_t)'$ ,  $\alpha$  is  $2 \times 1$ ,  $\beta$  is  $2 \times 1$ ,  $\mu_1$  is scalar,  $\mu_2$  is scalar, and  $\beta_3$  is scalar. For simplicity, it is assumed that structural break appears only in the long-run parameters  $\beta$  and  $\mu$  and has no effect on the short-run parameters<sup>50</sup>  $\alpha$  and  $\Gamma_i$ .

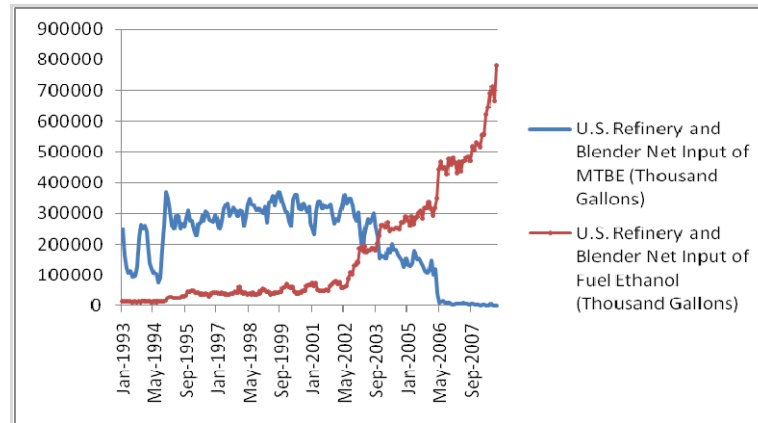


Figure 25. Blenders' use of MTBE and ethanol (January 1993 – October 2008)

However, maximizing the log-likelihood function for (15) using Andrade and Bruneau (2000) approach yields an estimate of  $\mu_2$  that is not significantly different from zero. As a result,  $sbd_{may07,t}$  is dropped from (15) and the model becomes

<sup>50</sup> Serra et al. (2008) assumes otherwise in their study of the corn-ethanol-oil price relationships within the US ethanol industry. Andrade and Bruneau (2000), in a multivariate analysis of a cointegrated vectorial autoregressive model with structural breaks affecting the cointegrating vectors, allows for a possible regime shift in both the short-run and the long-run parameters.

$$\Delta Y_t = \alpha \{ \beta' Y_{t-1} + \beta_3 (sbd_{may07,t} * pg_t) + \mu_1 \} + \sum_{i=1}^{12} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \quad (16)$$

Andrade and Bruneau (2000) shows that the log-likelihood function for (16) can also be concentrated in form (3), however, with the following notations differently defined

$$Z'_{0t} = \Delta Y'_t = (\Delta pe_t \quad \Delta pg_t) \quad (17)$$

$$Z'_{1t} = (pe_{t-1} \quad pg_{t-1} \quad 1 \quad sbd_{may07,t-1} * pg_{t-1})$$

$$Z'_{2t} = (\Delta pe_{t-1} \quad \Delta pg_{t-1} \quad \dots \quad \Delta pe_{t-12} \quad \Delta pg_{t-12})$$

Solving the generalized eigenvalue problem<sup>51</sup> (6) noting that  $S_{ij}, i, j \in \{0,1\}$  are as defined in (5),  $R_{it}, i \in \{0,1\}$  are as defined in (3), and  $Z_{jt}, j \in \{0,1,2\}$  are as defined in (17); and imposing (7), one obtains the estimates of the cointegrating parameters in (16). Based on these estimates, the long-run relationship between the ethanol price and the gasoline price<sup>52</sup> is

$$pe_t = \underset{(0.0774)}{0.9188} pg_t - \underset{(0.0588)}{0.1977} (sbd_{may07,t} * pg_t) + \underset{(0.0740)}{0.5697} \quad (18)$$

(18) shows how the ethanol price tracks the gasoline price. Alternatively, it provides an estimate of the long-run equilibrium ethanol price and breaks down into the pre-May 2007 equation

$$pe_t = 0.9188 pg_t + 0.5697 \text{ for } t < may07 \quad (19)$$

and the post-May 2007 equation

$$pe_t = 0.7211 pg_t + 0.5697 \text{ for } t \geq may07 \quad (20)$$

If ethanol was valued based on its energy content (i.e. which is roughly two-thirds that of gasoline) alone, the ethanol price would have to be roughly two-thirds the price of gasoline for blenders to be indifferent between blending any amount of ethanol less than the 10-percent-ethanol-90-percent-gasoline blending ratio. In **Figure 23**,  $\overline{pe}$  would have to be roughly two-thirds the price of gasoline if ethanol was valued based on its energy content

<sup>51</sup> *vec* command in Stata 10 can only be used in the absence of structural break.

<sup>52</sup> Standard errors of the coefficients are in parentheses and are obtained from (9).

alone. However, because ethanol is also used as an octane enhancer<sup>53</sup>, the ethanol price  $\overline{pe}$  may be above two-thirds the price of gasoline. Note that the octane enhancing value of ethanol is partially offset by the negative value of ethanol's high vapor pressure and water absorption ("Review of market," 2000). Tax benefits and incentive programs, such as the federal Volumetric Ethanol Excise Tax Credit (VEETC) of \$0.45 per gallon of ethanol blended into gasoline<sup>54</sup> and varying supplemental state subsidies<sup>55</sup> for ethanol use, may also add to the positive margin between the ethanol price  $\overline{pe}$  and the fuel value of ethanol. As long as ethanol is used mainly as a gasoline volume extender and its use does not exceed the 10-percent-ethanol-90-percent-gasoline blending ratio, the ethanol price should be close to the fuel value of ethanol plus the value of the tax benefits and incentive programs in place. The constant term in the long-run equilibrium ethanol price equation (18) captures the value of the tax benefits and incentive programs and is equal to \$0.57 per gallon of ethanol in both the pre-May 2007 and the post-May 2007 periods. Since the ethanol price in excess of the federal and state tax credits is shown in (19) to be significantly above the fuel value of ethanol and historical fuel ethanol use accounted for less than 10 percent of the finished fuel in the pre-May 2007 period, (19) is simply an estimate of the long-run equilibrium ethanol price and not an estimate of the perfectly elastic derived demand for fuel ethanol  $\overline{pe}$ . It also follows that the demand for fuel ethanol in the pre-May 2007 period is largely governed by government regulations such as the 1990 CAAA oxygenate requirements for oxygenated and reformulated gasoline. The low gasoline price provides little incentive for blenders' use of ethanol as a gasoline volume extender despite the tax benefits and incentive programs in place. The estimate of the coefficient of the gasoline price in (19) of 0.92 is much higher

<sup>53</sup> According to the National Renewable Energy Laboratory (NREL), 10% blend of ethanol in gasoline raises the octane number by 2.5 points. The value of an octane gallon (i.e. the value in excess of the price of a gallon of regular gasoline that a refiner would pay for a gallon of gasoline blending component having a blending octane number one number higher than the refinery's average output) is shown to be \$0.0071 - \$0.0143 per octane gallon.

<sup>54</sup> The federal tax credit was \$0.54 per gallon for ethanol use prior to January 1, 2005 and was \$0.51 per gallon for ethanol use from January 1, 2005 to December 31, 2008. On January 1, 2009, it was reduced further to \$0.45 per gallon for ethanol use.

<sup>55</sup> Several states provide reductions or exemptions for ethanol from motor fuel excise or sales taxes, the largest of which appear to be in Hawaii, Illinois, Indiana, and Iowa (Koplow & Steenblik, 2008). See [http://www.afdc.energy.gov/afdc/ethanol/incentives\\_laws.html](http://www.afdc.energy.gov/afdc/ethanol/incentives_laws.html) for more details.



than Higgins et al.'s (2006) estimate of the coefficient of the gasoline price<sup>56</sup> of 0.08. The difference may come from Higgins et al.'s inclusion of the period prior to 1995 in their study of the cointegrating relationship between the ethanol price and the gasoline price over the period of June 1989 to August 2005. Prior to 1995, monthly fuel ethanol use is shown to be most of the time below 100 million gallons in Error! Reference source not found.. Tyner's (2007) estimates of the relationship between the ethanol price and the gasoline price for the entire period 1982 – 2006 and for the separate periods 1982 – 2001 and 2002 – 2006 are also different from the estimate of the long-run equilibrium ethanol price (19). However, these estimates are not cointegrating relationships.

On the other hand, since the ethanol price in excess of the federal and state tax credits is shown in (20) to be close to the fuel value of ethanol and historical fuel ethanol use accounted for less than 10 percent of the finished fuel in the post-May 2007 period, (20) provides not only an estimate of the long-run equilibrium ethanol price but also an estimate of the perfectly elastic derived demand for fuel ethanol  $\overline{pe}$ . The significantly higher gasoline price in the post-May 2007 period may have increased blenders' use of ethanol as a gasoline volume extender. The increased use of ethanol as a gasoline volume extender, together with the MTBE phase-out and the repeal of the oxygenate requirement for reformulated gasoline (which made ethanol the main fuel additive for increasing octane), may have caused the ethanol price in excess of the tax credits in the post-May 2007 period to become close to the fuel value of ethanol. Given that the market operates in the perfectly elastic portion (20) of the derived demand for fuel ethanol, it follows that changes in the market equilibrium price of ethanol in the post-May 2007 period come primarily from changes in the derived demand for fuel ethanol.

## 6 Conclusions

This paper tests the hypothesis that the derived demand for fuel ethanol is perfectly elastic to see whether a long-run relationship exists between the ethanol price and the gasoline price over the period from January 1995 to October 2008. A VECM is used so that the Johansen and Juselius multivariate cointegration methodology is carried out. The

<sup>56</sup> No estimate of the constant term in the cointegrating relationship is provided by Higgins et al. (2006).

cointegration analysis finds no cointegration between the ethanol price and the gasoline price. However, this failure to find cointegration does not necessarily imply the lack of long-run relationship between the ethanol price and the gasoline price. The ethanol price and the gasoline price may be cointegrated in the sense that their linear combination (the cointegrating vector) is stationary but has shifted at one point in time (Gregory & Hansen, 1996). In fact, the GH residual-based tests for cointegration in models with regime shifts indicate that the long-run relationship between the ethanol price and the gasoline price exists with a possible structural break at the estimated break point May 2007. The MTBE phase-out, the repeal of the oxygenate requirement for reformulated gasoline, and the significantly higher gasoline price in the post-May 2007 period may have caused this break in the cointegrating relationship. Based on the obtained cointegrating relationship, the demand for fuel ethanol in the pre-May 2007 period is largely governed by government regulations such as the 1990 CAAA oxygenate requirements for oxygenated and reformulated gasoline, while the demand for fuel ethanol in the post-May 2007 period is perfectly elastic. Given that the market operates in the perfectly elastic portion of the derived demand for fuel ethanol, it follows that changes in the market equilibrium price of ethanol come primarily from changes in the derived demand for fuel ethanol. The linkage between the ethanol price and the gasoline price should prove useful for decision makers involved in the industry and policy makers in formulating biofuel and energy policy.

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## CHAPTER 4. WATER RECYCLING IN FUEL ETHANOL PLANT

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### Abstract

Among options available to help reduce some of the ethanol pressure on the (ground) water resources is water recycling in ethanol plants. Although modern ethanol plants possess sophisticated water treatment techniques for water recycling, water recycling is done only when it is cheaper than obtaining water from the outside source. Since water recycling can lower the cost of production, it may adversely induce production expansion and lead to more outside water being used by the plants. This paper examines the conditions under which this possibility occurs.

### 1 Introduction

Over the past decade, US fuel ethanol industry has been growing at a rapid rate, with more than 9.2 billion gallons of fuel ethanol produced in 2008 as compared with 1.1 to 1.47 billion gallons produced<sup>57</sup> annually between 1992 and 1997. The number of production facilities has grown from 50 operating plants in 1999 (Young & Briggs, 2007) to about 145 plants currently operating in 26 states (Wilkins, 2008). The growing demand for fuel ethanol comes largely from the need for gasoline substitutes as gasoline prices increase and crude oil supplies become less available; from the need for cleaner-burning fuel as concerns over carbon emissions intensify; and from the need for alternative fuel oxygenates to replace Methyl Tertiary Butyl Ether (MTBE) recognized by the US Environmental Protection Agency (EPA) as a potential ground water pollutant. Federal and state incentive programs, such as tax credits; oxygenate requirements; and renewable fuel standard (RFS) mandating biofuel use, were set up to promote fuel ethanol use. Advancements in the production technology also play an important role in the industry expansion.

<sup>57</sup> See [http://tonto.eia.doe.gov/dnav/pet/hist/m\\_epooxe\\_yop\\_nus\\_1m.htm](http://tonto.eia.doe.gov/dnav/pet/hist/m_epooxe_yop_nus_1m.htm) for more details.

Currently, ethanol is primarily produced from corn<sup>58</sup> in the US. In addition to water required to grow corn, a considerable amount of water is required to produce ethanol. According to Keeney and Muller (2006), a typical corn-based ethanol plant uses approximately 3.5 to 6 gallons of water for every gallon of ethanol produced. The considerable amount of water required for the production together with the dramatic industry expansion raises concerns about potential impacts on water supplies. The National Research Council (NRC), in its report on water implications of biofuel production<sup>59</sup>, addresses these water quantity concerns and identifies opportunities for water saving. Among the less costly and difficult methods to implement is water recycling in ethanol plants. Although modern ethanol plants possess sophisticated water treatment techniques for water recycling, water recycling is done only when it is cheaper than obtaining water from the outside source. Since water recycling can lower the cost of production, it may adversely induce production expansion and lead to more outside water being used by the plants. When this occurs, water recycling may no longer be beneficial in reducing the growing pressure from the ethanol plants on the water resources.

While rising attention has recently been directed toward water recycling in fuel ethanol plant, waste recycling has been the subject of many studies for quite some time. For instance, Anderson (1977) examines the principal economic arguments behind recycling incentives and via the use of econometric models of secondary material markets evaluates the projected impacts of recycling subsidies proposed in H.R. 148 and H.R. 10612 on the quantity of material recycled. Sigman (1995) provides a structural analysis of recycling policies possibly used to reduce environmental costs from waste disposal when direct disposal restrictions are difficult to enforce. A comparison of the policies in terms of their cost-effectiveness in reducing disposal is made using a partial equilibrium framework. An automobile-battery lead recycling example is used in examining the effects of these policies empirically. Hong and Adams (1999) investigates the effects of changes in solid waste

<sup>58</sup> Cellulose-based ethanol from trees, grasses, and crop wastes is not yet profitable on a large scale due to its current poor conversion efficiency. According to the National Research Council (NRC), the water requirements for cellulose-based ethanol production are projected to be approximately 2 to 6 gallons of water for every gallon of cellulose-based ethanol produced. However, less water may be required to grow cellulosic crops than to grow corn.

<sup>59</sup> See [http://books.nap.edu/openbook.php?record\\_id=12039&page=R1](http://books.nap.edu/openbook.php?record_id=12039&page=R1) for more details.

disposal service fees and household characteristics on solid waste generation and recycling rates under a block pricing system, using individual household data from Portland, Oregon. Households are assumed to first decide on weekly disposal volume for which to contract based on the expected amount of solid waste generated. They then decide on the amount of randomly produced total waste to recycle so that the weekly contracted volume is met. The ordered probit estimation method is used to estimate the volume choice model; while the two-stage estimation technique is used to estimate the demand equation for waste collection services.

However, these studies focus on recycling as a means of public waste reduction and do not deal explicitly with savings in natural resources used in the production of commodities which may arise from recycling. An exception is Smith (1972) who looks at a dynamic social optimization model of waste reuse where a representative household maximizes utility subject to a set of constraints over a continuous time, infinite horizon. Utility is obtained from commodity consumption whereas undesirable residues such as container units are created as consumption by-products. It is assumed that these container units can be recycled into the productive system at a cost in terms of a utility loss to the household. Container units not recycled are disposed and replaced by newly produced units. The prevailing stock of waste at each time enters the utility function as a bad and degrades at a certain rate. The stock of unrecovered raw material for commodities and containers is included in the model as another state variable to account for savings in natural resources which may arise from recycling.

This paper examines the possibility of water recycling in a corn-based fuel ethanol plant leading to more outside water being used by the plant due to production expansion. A static model of fuel ethanol production where a representative ethanol plant maximizes its profit subject to a set of constraints is used in making a comparison between the amount of outside water withdrawn by the plant when there is no water recycling and the amount of outside water withdrawn by the plant when there is water recycling. Alternative recycling incentives and their impacts on outside water use are also considered.

This paper is organized as follows. The next section provides background on ethanol production and water use. The third section presents a static model of a profit-maximizing

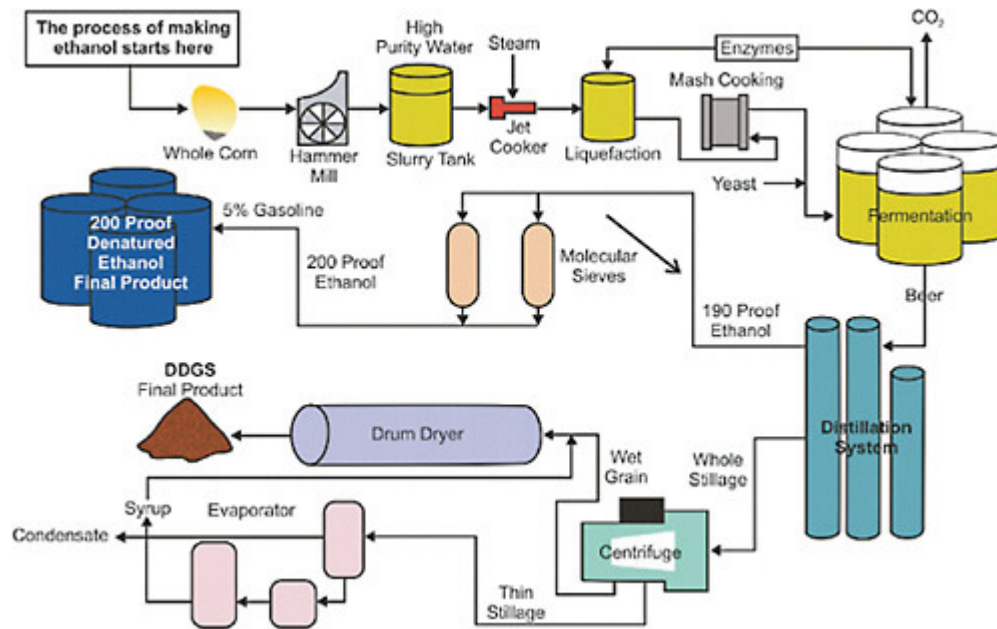


ethanol plant. The fourth section gives optimal rules for water use by the plant. The fifth section compares the amount of outside water withdrawn by the plant when there is no water recycling with the amount of outside water withdrawn by the plant when there is water recycling. The sixth section concludes.

## 2 Ethanol Production and Water Use

In the US, ethanol is primarily produced from corn via dry mill process (see **Figure 26**) where the entire corn kernels are ground into flour or cornmeal and processed without being separated into component parts.<sup>60</sup> The cornmeal is mixed with water and its pH adjusted. Enzymes are added to convert starch to sugar. The mixture is cooked at a high temperature before it is cooled and transferred to fermentation tanks. Yeast is added to convert sugar to ethanol and carbon dioxide. After 40 to 60 hours of fermentation, the fermented product is pumped into a distillation system where ethanol is separated from stillage (i.e. non-fermentable solids and water). The resulting 190-proof ethanol containing approximately 5% water is then passed through a molecular sieve system to remove the remaining water and to obtain the 200-proof anhydrous ethanol. A small amount of denaturant is added before the ethanol is shipped to gasoline terminals or retailers, making it unfit for human consumption and thus not subject to beverage alcohol tax. The stillage from the bottom of the distillation tanks is sent to centrifuges for separation into wet distillers grains (WDGs) and thin stillage. Some of the thin stillage may be routed back to the cooker for reuse as process water while the rest is concentrated via evaporation into high-protein-fat syrup. The syrup is added back to WDGs which may later be dried to obtain dried distillers grains (DDGs). Distillers grains can be used to feed livestock. Carbon dioxide released during fermentation can be captured and purified for use in carbonated beverages and flash-freezing applications.

<sup>60</sup> Another method used to produce corn ethanol is wet milling where the corn kernels are separated into component parts (e.g. starch, protein, germ, oil, kernel fibers, etc.) prior to fermentation. See “How ethanol is made” for more details.



**Figure 26. Corn-based dry milling ethanol production**

Sources: [http://books.nap.edu/openbook.php?record\\_id=12039&page=46](http://books.nap.edu/openbook.php?record_id=12039&page=46)

As can be seen, water is one of the key inputs in the ethanol production. In addition to being used as process water, it is also used as boiler water and cooling water in the plant's utility systems. Water can come into the plant from either nearby rivers or ground water sources. The plant may rely primarily on ground water which is typically readily available and of higher quality than surface water<sup>61</sup> (Mowbray & Hume, 2007). Thus, ethanol plants can present local (or regional) water problems if they are located where the water resources are already under stress. Many options are available to help reduce water consumption by an ethanol plant. They include switching from a wet cooling tower to a dry cooling tower, installing a high efficiency dryer, and using alternative technologies to distillation such as pervaporation. Some of these new plant designs may even reduce water consumption down to 1.5 gallons per gallon of ethanol produced. However, they may be expensive to implement at existing plants.<sup>62</sup> Among the less costly and difficult methods to implement is water recycling where waste water from the production (i.e. used process water, used boiler

<sup>61</sup> Water quality can affect the cooking process and may account for scaling and corrosion in the plant's heating and cooling systems.

water, and used cooling water) is put back to use. Since water loss or consumptive use of water may occur from evaporation, not all of the water used in the production ends up as waste water which can be recycled or discharged.

### 3 Model

The paper focuses on a basic model of fuel ethanol production where an ethanol plant uses outside water  $ow$  and recycled water  $rw$  to produce ethanol.<sup>63</sup> Water recycling is considered as a continuous flow process. Outside water and recycled water are assumed perfect substitutes in the production function  $f(\cdot)$ .<sup>64</sup> Processes such as filtration and methanation may be used to remove leftover solids and organic matters in recycled water. An ethanol output is  $f(ow + rw)$ . As the plant may be subject to an output constraint such as nameplate capacity, it is assumed that the output  $f(ow + rw)$  cannot be greater than the amount  $\bar{Q}$ . Due to evaporation, not all inlet water ends up as waste water after use. So the amount of waste water available for recycling is  $\alpha ow$  where  $0 < \alpha < 1$ . Since it is assumed that recycled water is treated before it is reused, the marginal cost of water recycling (i.e. the marginal cost of treating recycled water) may be viewed as being constant and is denoted by  $C^{rw}$  in this case. Assuming waste water can either be recycled or discharged, the amount of waste water discharged is  $\alpha ow - rw$ . Since discharged water has to meet quality standards set by the governments, let's assume that it is treated prior to being released into the streams at a per unit cost  $C^d$ .

Let  $P^e$  be a per unit price of ethanol; let  $P^{ow}$  be a per unit price of outside water,<sup>65</sup> and let all the markets be perfectly competitive, the profit maximization problem of the plant may be expressed as follows:

<sup>62</sup> See Jessen (2007) for more details.

<sup>63</sup> Other factors of production can be added to the model without changing the analysis.

<sup>64</sup> The quality difference between outside water and recycled water may be accounted for in the model by the use of different extraction costs for different sources of water.

<sup>65</sup>  $P^{ow}$  could also be thought of as the price of water obtained from city utilities.

$$\max_{ow, rw} P^e f(ow + rw) - P^{ow} ow - C^{rw} rw - C^d \{\alpha ow - rw\} \quad (1)$$

subject to  $rw \leq \alpha ow$

$$ow, rw \geq 0$$

$$f(ow + rw) \leq \bar{Q}$$

#### 4 Optimal Rules

The Lagrangian for system (1) is:

$$L = \left\{ \begin{array}{l} P^e f(ow + rw) - P^{ow} ow - C^{rw} rw - C^d \{\alpha ow - rw\} \\ + \lambda \{\alpha ow - rw\} + \beta \{\bar{Q} - f(ow + rw)\} \end{array} \right\} \quad (2)$$

where  $\lambda, \beta \geq 0$ . The necessary conditions for an optimal solution are:

$$\frac{\partial L}{\partial ow} = P^e \frac{\partial f(ow+rw)}{\partial ow} - P^{ow} - C^d \alpha + \lambda \alpha - \beta \frac{\partial f(ow+rw)}{\partial ow} \leq 0, ow \geq 0, \frac{\partial L}{\partial ow} ow = 0 \quad (3)$$

$$\frac{\partial L}{\partial rw} = P^e \frac{\partial f(ow+rw)}{\partial rw} - C^{rw} + C^d - \lambda - \beta \frac{\partial f(ow+rw)}{\partial rw} \leq 0, rw \geq 0, \frac{\partial L}{\partial rw} rw = 0 \quad (4)$$

$$\frac{\partial L}{\partial \lambda} = \alpha ow - rw \geq 0, \lambda \geq 0, \frac{\partial L}{\partial \lambda} \lambda = 0 \quad (5)$$

$$\frac{\partial L}{\partial \beta} = \bar{Q} - f(ow + rw) \geq 0, \beta \geq 0, \frac{\partial L}{\partial \beta} \beta = 0 \quad (6)$$

Since recycled water  $rw$  cannot exceed waste water from the production  $\alpha ow$ , the plant always uses outside water (i.e.  $ow > 0$ ), or else no ethanol would be produced. As a result, the following is obtained from (3):

$$P^e \frac{\partial f(ow+rw)}{\partial ow} + \lambda \alpha = P^{ow} + C^d \alpha + \beta \frac{\partial f(ow+rw)}{\partial ow} \quad (7)$$

(7) has that the marginal benefits of using outside water must equal the marginal costs. The marginal benefits break down into the marginal value product of outside water ( $P^e \frac{\partial f(ow+rw)}{\partial ow}$ ) and the benefit of having  $\alpha$  more units of waste water to recycle ( $\lambda \alpha$ ). The marginal costs break down into the per unit cost of outside water ( $P^{ow}$ ), the cost of having  $\alpha$  more units of waste water to discharge ( $C^d \alpha$ ), and the cost of using outside water associated with a  $\frac{\partial f(ow+rw)}{\partial ow}$  unit increase in output arising from the quantity constraint  $f(ow + rw) \leq \bar{Q}$  ( $\beta \frac{\partial f(ow+rw)}{\partial ow}$ ).

If  $0 < rw < \alpha ow$ , one has from (5) that  $\lambda = 0$ . From (4), the following is obtained:

$$P^e \frac{\partial f(ow+rw)}{\partial rw} + C^d = C^{rw} + \beta \frac{\partial f(ow+rw)}{\partial rw} \quad (8)$$

(8) has that the marginal benefits of using recycled water must equal the marginal costs. The marginal benefits break down into the marginal value product of recycled water ( $P^e \frac{\partial f(ow+rw)}{\partial rw}$ ) and the per unit cost of discharge saved by recycling an additional unit of waste water ( $C^d$ ). The marginal costs break down into the per unit cost of water recycling ( $C^{rw}$ ) and the cost of using recycled water associated with a  $\frac{\partial f(ow+rw)}{\partial rw}$  unit increase in output arising from the quantity constraint  $f(ow+rw) \leq \bar{Q}$  ( $\beta \frac{\partial f(ow+rw)}{\partial rw}$ ). It is then obtained from (7), (8), and

$\frac{\partial f(ow+rw)}{\partial ow} = \frac{\partial f(ow+rw)}{\partial rw}$  that:

$$C^{rw} - C^d = P^{ow} + C^d \alpha \quad (9)$$

(9) states that the net cost of recycled water, i.e. the marginal cost of water recycling ( $C^{rw}$ ) minus the discharge cost saved by recycling water ( $C^d$ ), must equal the net cost of outside water, i.e. the per unit cost of outside water ( $P^{ow}$ ) plus the cost of having  $\alpha$  more units of waste water to discharge ( $C^d \alpha$ ), for the plant to recycle some of its waste water.

If  $rw = 0$ , one has from (5) that  $\lambda = 0$ . From (4), the following is obtained:

$$P^e \frac{\partial f(ow)}{\partial rw} + C^d \leq C^{rw} + \beta \frac{\partial f(ow)}{\partial rw} \quad (10)$$

(10) has that the marginal benefits of using recycled water must be less than or equal to the marginal costs. It is then obtained from (7), (10),  $\frac{\partial f(ow+rw)}{\partial ow} = \frac{\partial f(ow+rw)}{\partial rw}$ , and  $rw = 0$  that:

$$C^{rw} - C^d \geq P^{ow} + C^d \alpha \quad (11)$$

(11) states that the net cost of recycled water must be greater than or equal to the net cost of outside water for recycled water to not be used.

If  $rw = \alpha ow$ , one has from (5) that  $\lambda \geq 0$ . From (4), the following is obtained:

$$P^e \frac{\partial f(ow+\alpha ow)}{\partial rw} + C^d = C^{rw} + \lambda + \beta \frac{\partial f(ow+\alpha ow)}{\partial rw} \quad (12)$$

Note that  $\lambda$  is the shadow price of waste water available for recycling. (12) has that the marginal benefits of using recycled water must equal the marginal costs. It is then obtained from (7), (12),  $\frac{\partial f(ow+rw)}{\partial ow} = \frac{\partial f(ow+rw)}{\partial rw}$ , and  $rw = \alpha ow$  that:

$$(1 + \alpha)\lambda = (P^{ow} + C^d \alpha) - (C^{rw} - C^d) \geq 0 \quad (13)$$

(13) implies the following:

$$C^{rw} - C^d \leq P^{ow} + C^d \alpha \quad (14)$$

(14) states that the net cost of recycled water must be less than or equal to the net cost of outside water for the plant to recycle all of its waste water.

In order to examine whether water recycling leads to a reduction in the amount of outside water withdrawn by the plant, optimal rules are needed for outside water use before and after water recycling. Section 4.1 and 4.2 below provide these rules.

#### 4.1 Ethanol Plant with No Water Recycling

Let's first assume the plant utilizes no water recycling ( $rw = 0$ ),<sup>66</sup> that is initially assuming the net cost of recycled water is greater than the net cost of outside water as in (11). If the output constraint holds with strict inequality, (6) implies  $\beta = 0$ . As a result, (7) becomes the following:

$$P^e \frac{\partial f(ow^{*A})}{\partial ow} = P^{ow} + C^d \alpha \text{ where } f(ow^{*A}) < \bar{Q} \quad (15)$$

(15) is an implicit equation for  $ow^{*A}$ . Superscript \*A denotes optimality in this case.

However, if the output constraint holds with equality, one has from (7) that:

$$P^e \frac{\partial f(ow^{*B})}{\partial ow} = P^{ow} + C^d \alpha + \beta^{*B} \frac{\partial f(ow^{*B})}{\partial ow} \text{ where } ow^{*B} = f^{-1}(\bar{Q}) \quad (16)$$

Since the water use is fixed at  $ow^{*B} = f^{-1}(\bar{Q})$ , (16) provides an implicit equation for  $\beta^{*B} > 0$ . Superscript \*B denotes optimality in this case.

#### 4.2 Ethanol Plant with Water Recycling

Other things being equal, since it is assumed in section 3.1 that  $C^{rw} - C^d \geq P^{ow} + C^d \alpha$ , the plant has no reason to recycle water unless there is a decrease in the net cost of recycled water or an increase in the net cost of outside water. The plant may not take into account other uses of water when deciding how much outside water to withdraw. As a result,  $ow^{*A}$  and  $ow^{*B}$  in section 4.1 may not be socially optimal. **Figure 27** and **Figure 28** illustrate this externality problem<sup>67</sup> when the plant uses  $ow^{*A} < f^{-1}(\bar{Q})$  and  $ow^{*B} = f^{-1}(\bar{Q})$  of outside water respectively in the absence of water recycling. *MB* is the marginal benefit of using water to produce ethanol, *MPC* is the marginal private cost, and *MSC* is the marginal social cost.  $w^S$  is the socially optimal use of water to produce ethanol

<sup>66</sup> In other words, let's assume the plant discharges all of its waste water and that discharged water meets quality standards set by the EPA.

when other uses of water are also taken into account. The difference between  $MSC$  and  $MPC$  is the user cost of (ground) water in ethanol production (i.e. the cost of using water to produce ethanol in terms of forgone other uses of water).

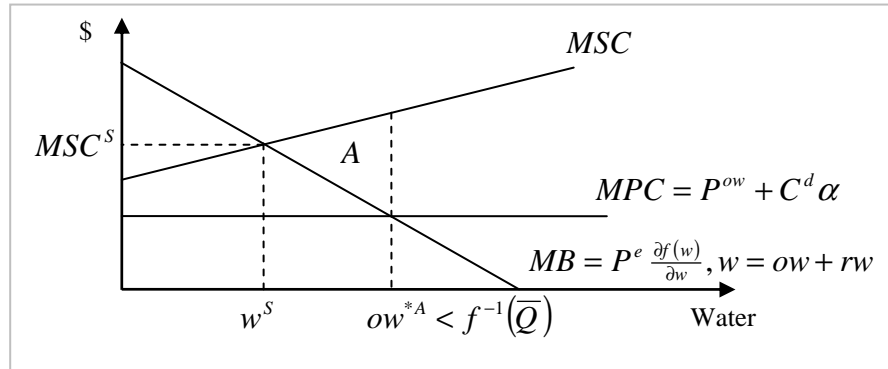


Figure 27. Externality problem when the plant uses  $ow^{*A} < f^{-1}(\bar{Q})$  on its own

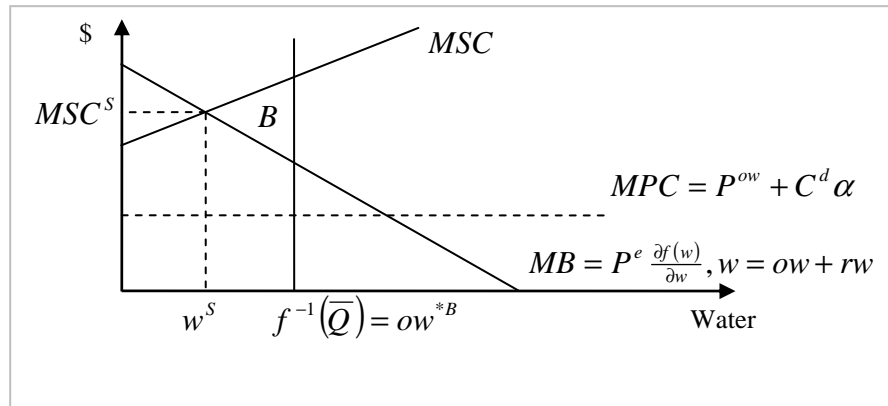


Figure 28. Externality problem when the plant uses  $ow^{*B} = f^{-1}(\bar{Q})$  on its own

By moving to the internalized solution  $w^S$ , the community gains an area  $A$  more than the plant loses in **Figure 27** and an area  $B$  in **Figure 28**.

The government, in attempt to alleviate some of the ethanol pressure on the (ground) water resources, may provide incentives to reduce outside water use.<sup>68</sup>

- Since the plant tends to overuse outside water, the government may impose a

<sup>67</sup> See Just, Hueth, and Schmitz (2004) for discussion of externalities and the approach of internalization.

- Pigouvian tax on outside water use.<sup>69</sup> Let  $T$  be the tax on outside water. Since  $T$  raises the marginal private cost of using outside water  $P^{ow} + C^d \alpha$ , the plant produces less ethanol and therefore uses less outside water. The amount of outside water used by the plant can be reduced even further due to water recycling.
- In case a tax cannot be imposed on outside water use (e.g. due to cities competing for ethanol plants), the government may subsidize water recycling.<sup>70</sup> Let  $S$  be the subsidy for water recycling. Since  $S$  leaves the marginal private cost of using outside water  $P^{ow} + C^d \alpha$  unchanged while lowering the net cost of recycled water  $C^{rw} - C^d$  and if the marginal benefit  $P^e \frac{\partial f(w)}{\partial w}$  remains the same, the plant produces the same amount of ethanol but uses less outside water due to water recycling. In this sense, a subsidy for water recycling tends to be less efficient than a tax on outside water use.

Depending on the magnitude of the policy used by the government, the plant may recycle some or all of its waste water:

#### Ethanol Plant Recycling Some of Its Waste Water

Let  $\Theta$  be such that  $C^{rw} - C^d - \Theta = P^{ow} + C^d \alpha$  where  $\Theta = T$  if the government taxes outside water use and  $\Theta = S$  if the government subsidizes water recycling. Since the net cost of recycled water now equals the net cost of outside water, the plant recycles some of its waste water in this case. One then has from (5) that  $\lambda = 0$ . Let's assume diminishing marginal productivity of water (i.e.  $\frac{d^2 f(w)}{dw^2} < 0, w = ow + rw$ ). If the output constraint holds with strict inequality in section 3.1, depending on what type of policy  $\Theta$  is, (7) becomes the following:

- If  $\Theta$  is a per unit tax on outside water,

$$P^e \frac{\partial f(ow^{**A1} + rw^{**A1})}{\partial ow} = P^{ow} + C^d \alpha + T \quad (17)$$

<sup>68</sup> See Baumol and Oates (1988) for the efficiency properties of effluent fees.

<sup>69</sup> See Miranda, Everett, Blume, and Roy (1994) for the use of market-based incentives to encourage both source reducing and waste diversion in dealing with residential municipal solid waste.

<sup>70</sup> See Anderson (1977) and Sigman (1995) for the use of recycling subsidies when direct restrictions on disposal are difficult to enforce.



where  $f(ow^{**A1} + rw^{**A1}) < \bar{Q}$  and

superscript **\*\*A1** denotes optimality in this case.

- If  $\Theta$  is a per unit subsidy for water recycling,

$$P^e \frac{\partial f(ow^{**A2} + rw^{**A2})}{\partial ow} = P^{ow} + C^d \alpha \quad (18)$$

where  $f(ow^{**A2} + rw^{**A2}) < \bar{Q}$  and

superscript **\*\*A2** denotes optimality in this case.

(17) and (18) are implicit equations for  $ow^{**A1} + rw^{**A1}$  and  $ow^{**A2} + rw^{**A2}$  respectively.

Since  $\Theta$  raises the marginal costs of using outside water  $P^{ow} + C^d \alpha$  in (17) by the amount  $T$  and leaves  $P^{ow} + C^d \alpha$  unchanged in (18), the output constraint continues to hold with strict inequality in the presence of water recycling regardless of the policy choice  $\Theta$ .

However, if the output constraint holds with equality in section 3.1, depending on what type of policy  $\Theta$  is and the magnitude of  $\Theta$ , the output constraint may not continue to hold with equality in the presence of water recycling. One has from (7) that:

- If  $\Theta$  is a per unit tax on outside water,

$$\triangleright P^e \frac{\partial f(ow^{**B1a} + rw^{**B1a})}{\partial ow} = P^{ow} + C^d \alpha + T \quad (19)$$

where  $f(ow^{**B1a} + rw^{**B1a}) < \bar{Q}$ ,  $T > \beta^{*B} \frac{\partial f(ow^{*B})}{\partial ow}$ , and

superscript **\*\*B1a** denotes optimality in this case.

$$\triangleright P^e \frac{\partial f(ow^{**B1b} + rw^{**B1b})}{\partial ow} = P^{ow} + C^d \alpha + T + \beta^{**B1b} \frac{\partial f(ow^{**B1b} + rw^{**B1b})}{\partial ow} \quad (20)$$

where  $ow^{**B1b} + rw^{**B1b} = f^{-1}(\bar{Q})$ ,  $T < \beta^{*B} \frac{\partial f(ow^{*B})}{\partial ow}$ , and

superscript **\*\*B1b** denotes optimality in this case.

- If  $\Theta$  is a per unit subsidy for water recycling,

$$P^e \frac{\partial f(ow^{**B2} + rw^{**B2})}{\partial ow} = P^{ow} + C^d \alpha + \beta^{**B2} \frac{\partial f(ow^{**B2} + rw^{**B2})}{\partial ow} \quad (21)$$

where  $ow^{**B2} + rw^{**B2} = f^{-1}(\bar{Q})$  and

superscript **\*\*B2** denotes optimality in this case.

(19) is an implicit equation for  $ow^{**B1a} + rw^{**B1a}$ . Since the total water use is fixed at  $f^{-1}(\bar{Q})$ , (20) and (21) provide implicit equations for  $\beta^{**B1b} > 0$  and  $\beta^{**B2} > 0$ .

### Ethanol Plant Recycling All of Its Waste Water

Let  $\Theta$  be such that  $C^{rw} - C^d - \Theta < P^{ow} + C^d\alpha$  where  $\Theta = \tau$  if the government taxes outside water use and  $\Theta = S$  if the government subsidizes water recycling. Since the net cost of recycled water is now lower than the net cost of outside water, the plant recycles all of its waste water in this case. One then has  $rw = \alpha ow$ . As  $\Theta$  raises the opportunity cost of not recycling water, the shadow price of waste water available for recycling becomes

$$\lambda = \frac{(P^{ow} + C^d\alpha) - (C^{rw} - C^d - \Theta)}{(1+\alpha)} > 0. \text{ If the output constraint holds with strict inequality in section 3.1,}$$

depending on what type of policy  $\Theta$  is and whether the output constraint continues to hold with strict inequality in the presence of water recycling, (7) becomes the following:

- If  $\Theta$  is a per unit tax on outside water,

$$P^e \frac{\partial f(ow^{***A1} + rw^{***A1})}{\partial ow} + \left( \frac{P^{ow} + C^d\alpha - C^{rw} + C^d + T}{1+\alpha} \right) \alpha = P^{ow} + C^d\alpha + T \quad (22)$$

where  $f(ow^{***A1} + rw^{***A1}) < \bar{Q}$  and

superscript \*\*\*A1 denotes optimality in this case.

- If  $\Theta$  is a per unit subsidy for water recycling,

$$\triangleright P^e \frac{\partial f(ow^{***A2a} + rw^{***A2a})}{\partial ow} + \left( \frac{P^{ow} + C^d\alpha - C^{rw} + C^d + S}{1+\alpha} \right) \alpha = P^{ow} + C^d\alpha \quad (23)$$

where  $f(ow^{***A2a} + rw^{***A2a}) < \bar{Q}$  and

superscript \*\*\*A2a denotes optimality in this case.

$$\triangleright \left\{ \begin{array}{l} P^e \frac{\partial f(ow^{***A2b} + rw^{***A2b})}{\partial ow} + \left( \frac{P^{ow} + C^d\alpha - C^{rw} + C^d + S}{1+\alpha} \right) \alpha \\ = P^{ow} + C^d\alpha + \beta^{***A2b} \frac{\partial f(ow^{***A2b} + rw^{***A2b})}{\partial ow} \end{array} \right\} \quad (24)$$

where  $ow^{***A2b} = \frac{f^{-1}(\bar{Q})}{1+\alpha}$ ,  $rw^{***A2b} = \frac{\alpha f^{-1}(\bar{Q})}{1+\alpha}$ , and

superscript \*\*\*A2b denotes optimality in this case.

Since the plant recycles all of its waste water in this case, (22) and (23) are implicit equations

for  $ow^{***A1}$  and  $ow^{***A2a}$  respectively. (24) provides an implicit equation for  $\beta^{***A2b} > 0$

respectively as the total water use is fixed at  $f^{-1}(\bar{Q})$ . Since the tax on outside water raises

the net marginal cost of using outside water in (22) by the amount  $T - \left( \frac{P^{ow} + C^d\alpha - C^{rw} + C^d + T}{1+\alpha} \right) \alpha$ ,

the output constraint continues to hold with strict inequality in this case. The subsidy for

water recycling, on the other hand, lowers the net marginal cost of using outside water. So it is possible that the output constraint holds instead with equality. To see exactly when the output constraint continues to hold with strict inequality in the presence of water recycling, functional forms may be needed in this case.

However, if the output constraint holds with equality in section 3.1, depending on what kind of policy  $\Theta$  is, one has from (7) that:

- If  $\Theta$  is a per unit tax on outside water,

$$P^e \frac{\partial f(ow^{***B1} + rw^{***B1})}{\partial ow} + \left( \frac{P^{ow} + C^d \alpha - C^{rw} + C^d + T}{1 + \alpha} \right) \alpha = P^{ow} + C^d \alpha + T \quad (25)$$

where  $f(ow^{***B1} + rw^{***B1}) < \bar{Q}$  and

superscript \*\*\*B1 denotes optimality in this case.

- If  $\Theta$  is a per unit subsidy for water recycling,

$$\left\{ \begin{aligned} & P^e \frac{\partial f(ow^{***B2} + rw^{***B2})}{\partial ow} + \left( \frac{P^{ow} + C^d \alpha - C^{rw} + C^d + S}{1 + \alpha} \right) \alpha \\ & = P^{ow} + C^d \alpha + \beta^{***B2} \frac{\partial f(ow^{***B2} + rw^{***B2})}{\partial ow} \end{aligned} \right\} \quad (26)$$

where  $ow^{***B2} = \frac{f^{-1}(\bar{Q})}{1 + \alpha}$ ,  $rw^{***B2} = \frac{\alpha f^{-1}(\bar{Q})}{1 + \alpha}$ , and

superscript \*\*\*B2 denotes optimality in this case.

Since the plant recycles all of its waste water in this case, (25) is an implicit equation for  $ow^{***B1}$ . (26) provides an implicit equation for  $\beta^{***B2} > 0$  respectively as the total water use is fixed at  $f^{-1}(\bar{Q})$ .

## 5 Outside Water Use Before and After Water Recycling

With the optimal rules obtained, this section examines whether water recycling leads to a reduction in the amount of outside water withdrawn by the plant. A comparison is made between the amount of outside water withdrawn when there is no water recycling and the amount of outside water withdrawn when there is water recycling. Depending on whether the output constraint holds with strict inequality in the absence of water recycling and whether the plant recycle all of its waste water in the presence of  $\Theta$ , four different scenarios are considered:

**Scenario 1:** assuming the output constraint holds with strict inequality in the absence of water recycling, Scenario 1 compares the amount of outside water withdrawn when the plant recycles none of its waste water ( $ow^{*A} < f^{-1}(\bar{Q})$ ) with the amount of outside water withdrawn when the plant recycles some of its waste water ( $ow^{**Ai}, i = 1, 2$ ).

- If  $\Theta$  is a per unit tax on outside water, given diminishing marginal productivity of water (i.e.  $\frac{d^2 f(w)}{dw^2} < 0, w = ow + rw$ ), (15) and (17) yield  $ow^{**A1} + rw^{**A1} < ow^{*A}$  implying  $ow^{**A1} < ow^{*A}$ .
- If  $\Theta$  is a per unit subsidy for water recycling, (16) and (18) yield  $ow^{**A2} + rw^{**A2} = ow^{*A}$  implying  $ow^{**A2} < ow^{*A}$ .

Since the tax on outside water raises the marginal costs of using outside water, less outside water is used. The subsidy for water recycling, on the other hand, leaves the marginal costs of using outside water unchanged. So the total water use remains the same. However, because of water recycling, less outside water is used. As a result, water recycling does lead to a reduction in the amount of outside water withdrawn by the plant regardless of the policy choice  $\Theta$ .

**Scenario 2:** assuming the output constraint holds with equality in the absence of water recycling, Scenario 2 compares the amount of outside water withdrawn when the plant recycles none of its waste water ( $ow^{*B} = f^{-1}(\bar{Q})$ ) with the amount of outside water withdrawn when the plant recycles some of its waste water ( $ow^{**Bi}, i = 1a, 1b, 2$ ).

- If  $\Theta$  is a per unit tax on outside water and
  - if  $T > \beta^{*B} \frac{\partial f(ow^{*B})}{\partial ow}$ , the output constraint holds instead with strict inequality in the presence of water recycling. Given diminishing marginal productivity of water (i.e.  $\frac{d^2 f(w)}{dw^2} < 0, w = ow + rw$ ), (16) and (19) yield  $ow^{**B1a} + rw^{**B1a} < ow^{*B} = f^{-1}(\bar{Q})$  implying  $ow^{**B1a} < ow^{*B}$ .

➤ if  $T < \beta^{*B} \frac{\partial f(ow^{*B})}{\partial ow}$ , the output constraint continues to hold with equality in the presence of water recycling. **(16)** and **(20)** yield  $ow^{**B1b} + rw^{**B1b} = f^{-1}(\bar{Q}) = ow^{*B}$  implying  $ow^{**B1b} < ow^{*B}$ .

- If  $\Theta$  is a per unit subsidy for water recycling, since the output constraint continues to hold with equality in the presence of water recycling, **(16)** and **(21)** yield  $ow^{**B2} + rw^{**B2} = f^{-1}(\bar{Q}) = ow^{*B}$  implying  $ow^{**B2} < ow^{*B}$ .

Since the tax on outside water raises the marginal costs of using outside water, less outside water is used. The subsidy for water recycling, on the other hand, leaves the marginal costs of using outside water unchanged. So the total water use remains fixed at  $f^{-1}(\bar{Q})$ .

However, because of water recycling, less outside water is used. As a result, water recycling does lead to a reduction in the amount of outside water withdrawn by the plant regardless of the policy choice  $\Theta$  and whether the output constraint continues to hold with equality in the presence of water recycling.

**Scenario 3:** assuming the output constraint holds with equality in the absence of water recycling, Scenario 3 compares the amount of outside water withdrawn when the plant recycles none of its waste water ( $ow^{*B} = f^{-1}(\bar{Q})$ ) with the amount of outside water withdrawn when the plant recycles all of its waste water ( $ow^{***Bi}, i = 1, 2$ ).

- If  $\Theta$  is a per unit tax on outside water, since the output constraint holds instead with strict inequality in the presence of water recycling, **(16)** and **(25)** yield  $ow^{***B1} + rw^{***B1} < ow^{*B} = f^{-1}(\bar{Q})$  implying  $ow^{***B1} < ow^{*B}$ .
- If  $\Theta$  is a per unit subsidy for water recycling, since the output constraint continues to hold with equality in the presence of water recycling, **(16)** and **(26)** yield  $ow^{***B2} < ow^{*B}$ .

Since the tax on outside water raises the net marginal cost of using outside water, less outside water is used. The subsidy for water recycling, on the other hand, lowers the net marginal cost of using outside water. However, because of the output constraint, the total water use remains fixed at  $f^{-1}(\bar{Q})$  in the presence of the subsidy. Since the plant recycles all of its

waste water, less outside water is used. As a result, water recycling does lead to a reduction in the amount of outside water withdrawn by the plant regardless of the policy choice  $\Theta$  and whether the output constraint continues to hold with equality in the presence of water recycling.

**Scenario 4:** assuming the output constraint holds with strict inequality in the absence of water recycling, Scenario 4 compares the amount of outside water withdrawn when the plant recycles none of its waste water ( $ow^{*A} < f^{-1}(\bar{Q})$ ) with the amount of outside water withdrawn when the plant recycles all of its waste water ( $ow^{***Ai}, i = 1, 2a, 2b$ ).

- If  $\Theta$  is a per unit tax on outside water, since the output constraint continues to hold with strict inequality in the presence of water recycling, (15) and (22) yield  $ow^{***A1} + rw^{***A1} < ow^{*A}$  implying  $ow^{***A1} < ow^{*A}$ .
- If  $\Theta$  is a per unit subsidy for water recycling and
  - if the output constraint continues to hold with strict inequality in the presence of water recycling, given diminishing marginal productivity of water (i.e.  $\frac{d^2 f(w)}{dw^2} < 0, w = ow + rw$ ), (15) and (23) yield  $ow^{***A2a} + rw^{***A2a} > ow^{*A}$ . It is unclear whether  $ow^{***A2a} < ow^{*A}$ .
  - if the output constraint holds instead with equality in the presence of water recycling, it is unclear from (15) and (24) whether  $ow_t^{***A2b} < ow_t^{*A}$ .

As can be seen, water recycling does lead to a reduction in the amount of outside water withdrawn when the government taxes outside water use. This is because the tax raises the net marginal cost of using outside water. As for the water recycling subsidy, since the plant recycles all of its waste water in this case, a unit of outside water also possesses recycling value. The plant wanting to capture this recycling value may increase its use of outside water. As a result, it is unclear whether water recycling leads to a reduction in the amount of outside water withdrawn when the government subsidizes water recycling. To see exactly when water recycling leads to a reduction in the amount of outside water withdrawn when the government subsidizes water recycling, closed form solutions are needed so that  $ow^{*A}$

and  $ow^{***Ai}, i = 2a, 2b$  can be explicitly compared. Finding closed form solutions requires assuming a functional form for the production function of ethanol.

For illustrative purpose, let's assume an ethanol production function of the form:

$$\ln Q = \ln A + \gamma_1 \ln(ow + rw) + \gamma_2 \ln(ow + rw)^2 \quad (27)$$

where  $Q$  = ethanol output,  $A$  = technology parameter, and  $\gamma_1 + 2\gamma_2 < 1$  with diminishing marginal productivity. Written with  $Q$  on the left hand side, (27) becomes:

$$Q = A(ow + rw)^{\gamma_1} e^{\gamma_2 \ln(ow + rw)^2} = A(ow + rw)^{\gamma_1 + 2\gamma_2} \quad (28)$$

It then follows from (15) that the amount of outside water withdrawn by the plant in the absence of water recycling is:

$$ow^{*A} = \left[ \frac{P^e (\gamma_1 + 2\gamma_2) A}{P^{ow} + C^d \alpha} \right]^{\frac{1}{1 - \gamma_1 - 2\gamma_2}} \quad (29)$$

Depending on whether the output constraint continues to hold with strict inequality in the presence of water recycling, the amount of outside water withdrawn when the plant recycles all of its waste water in the presence of  $S$  is:

$$\text{From (23), } ow^{***A2a} = \left( \frac{1}{1 + \alpha} \right) \left[ \frac{P^e (\gamma_1 + 2\gamma_2) A}{P^{ow} + C^d \alpha - \lambda \alpha} \right]^{\frac{1}{1 - \gamma_1 - 2\gamma_2}} \text{ where } \lambda = \frac{(P^{ow} + C^d \alpha) - (C^{rw} - C^d - S)}{(1 + \alpha)} > 0 \quad (30)$$

$$\text{From (24), } ow^{***A2b} = \left( \frac{1}{1 + \alpha} \right) \left( \frac{\bar{Q}}{A} \right)^{\frac{1}{\gamma_1 + 2\gamma_2}} \quad (31)$$

Since by construction the output constraint holds with strict inequality in the absence of water recycling, one has from (29) that  $P^{ow} + C^d \alpha - P^e (\gamma_1 + 2\gamma_2) A \{f^{-1}(\bar{Q})\}^{\gamma_1 + 2\gamma_2 - 1} > 0$ . Note also that by construction  $[C^{rw} - C^d - P^{ow} - C^d \alpha] < S$ . As a result, the amount of outside water withdrawn when the plant recycles none of its waste water ( $ow^{*A}$ ) and the amount of outside water withdrawn when the plant recycles all of its waste water in the presence of  $S$  ( $ow^{***Ai}, i = 2a, 2b$ ) can now be explicitly compared.

- If  $S < [C^{rw} - C^d - P^{ow} - C^d \alpha] + \left[ \frac{P^{ow} + C^d \alpha - P^e (\gamma_1 + 2\gamma_2) A \{(1 + \alpha) f^{-1}(\bar{Q})\}^{\gamma_1 + 2\gamma_2 - 1}}{\alpha(1 + \alpha)^{-1}} \right]$ , the output constraint

continues to hold with strict inequality in the presence of water recycling. One has from (29) and (30) that water recycling leads to a reduction in the amount of outside water withdrawn by the plant when  $S$  falls within the below range:

$$[C^{rw} - C^d - P^{ow} - C^d \alpha] < S < \left\{ \begin{array}{l} [C^{rw} - C^d - P^{ow} - C^d \alpha] \\ + \min[\bar{S1}, \bar{S2}] \end{array} \right\} \quad (32)$$

$$\text{where } \bar{S1} = \frac{P^{ow} + C^d \alpha - P^e (\gamma_1 + 2\gamma_2) A \{(1+\alpha) f^{-1}(\bar{Q})\}^{\gamma_1 + 2\gamma_2 - 1}}{\alpha(1+\alpha)^{-1}}$$

$$\bar{S2} = (P^{ow} + C^d \alpha) \left( \frac{1+\alpha}{\alpha} \right) \left\{ 1 - \left( \frac{1}{1+\alpha} \right)^{1-\gamma_1 - 2\gamma_2} \right\}$$

- If  $S > [C^{rw} - C^d - P^{ow} - C^d \alpha] + \left[ \frac{P^{ow} + C^d \alpha - P^e (\gamma_1 + 2\gamma_2) A \{(1+\alpha) f^{-1}(\bar{Q})\}^{\gamma_1 + 2\gamma_2 - 1}}{\alpha(1+\alpha)^{-1}} \right]$ , the output

constraint holds instead with equality in the presence of water recycling. One has from (29) and (31) that water recycling leads to a reduction in the amount of outside

water withdrawn by the plant when  $\left( \frac{1}{1+\alpha} \right) \left( \frac{\bar{Q}}{A} \right)^{\frac{1}{\gamma_1 + 2\gamma_2}} < \left[ \frac{(1+r) P^e (\gamma_1 + 2\gamma_2) A}{(1+r) P^{ow} + C^d \alpha} \right]^{\frac{1}{1-\gamma_1 - 2\gamma_2}}$ .

As can be seen, because the subsidy for water recycling leaves the marginal costs of using outside water unchanged, the plant may want to increase its outside water use so that it would have more water to recycle and, therefore, can obtain more subsidy. As a result, a subsidy for water recycling that is too large may adversely induce production expansion and lead to more outside water being used by the plant.

## 6 Conclusions

Water recycling is among several options available to help reduce water consumption by an ethanol plant. In an attempt to reduce some of the ethanol pressure on the water resources, the government may provide incentives for the plant to reduce outside water use. Since the plant tends to overuse outside water, the government may impose a Pigouvian tax on outside water use. In case a tax cannot be imposed on outside water use, the government may subsidize water recycling. Depending on whether the output constraint holds with strict inequality in the absence of water recycling and whether the plant ends up recycle all of its waste water, a comparison is made under four different scenarios between the amount of outside water withdrawn by the plant when there is no water recycling and the amount of outside water withdrawn by the plant when there is water recycling.

Scenario 1 assumes the output constraint holds with strict inequality in the absence of water recycling and compares the amount of outside water withdrawn when the plant recycles none of its waste water with the amount of outside water withdrawn when the plant



recycles some of its waste water. Outside water use is reduced regardless of the policy choice  $\Theta$ . The tax on outside water reduces the total water use and at the same time induces water recycling. The subsidy for water recycling leaves the total water use unchanged while induces water recycling.

Scenario 2 assumes the output constraint holds with equality in the absence of water recycling and compares the amount of outside water withdrawn when the plant recycles none of its waste water with the amount of outside water withdrawn when the plant recycles some of its waste water. Outside water use is reduced regardless of the policy choice  $\Theta$  and whether the output constraint continues to hold with equality in the presence of water recycling. A large tax on outside water reduces the total water use and at the same time induces water recycling. A small tax on outside water and the subsidy for water recycling both leave the total water use unchanged while induces water recycling.

Scenario 3 assumes the output constraint holds with equality in the absence of water recycling and compares the amount of outside water withdrawn when the plant recycles none of its waste water with the amount of outside water withdrawn when the plant recycles all of its waste water. Outside water use is reduced regardless of the policy choice  $\Theta$  and whether the output constraint continues to hold with equality in the presence of water recycling. The tax on outside water reduces the total water use and at the same time induces water recycling. The subsidy for water recycling leaves the total water use unchanged because of the output constraint while induces water recycling.

Scenario 4 assumes the output constraint holds with strict inequality in the absence of water recycling and compares the amount of outside water withdrawn when the plant recycles none of its waste water with the amount of outside water withdrawn when the plant recycles all of its waste water. Water recycling does lead to a reduction in the amount of outside water withdrawn when the government taxes outside water use. However, it is not always the case that outside water use is reduced when the government subsidizes water recycling. Given the assumed ethanol production function, the subsidy for water recycling must be sufficiently small to reduce outside water use. A subsidy for water recycling that is too large may adversely induce production expansion and lead to more outside water being

used by the plant. When this occurs, water recycling may no longer be beneficial in reducing the growing pressure from the ethanol plants on the water resources.

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## CHAPTER 5. GENERAL CONCLUSIONS

This dissertation looks at backstop technology as a key to weak sustainability of commodity resources. As resources become scarcer, their price increases and signals a switch to a relatively more expensive renewable backstop technology. The first essay illustrates the role of backstop technology in sustainability of commodity resources through the use of renewable ground water example. In the renewable ground water example, both artificially recharged ground water and water from other cities act as backstop technologies for natural ground water. In the growing water demand model, the city may have to eventually resort to costly water from other cities to satisfy the excess demand. The artificial ground water recharge adds to the natural ground water. So it can help prolong the period of not having to purchase costly water from other cities. However, due to the costs associated with pumping water underground, the artificial ground water recharge is done only when the demand is sufficiently high. In the stochastic rainfall model, it may be relatively cheaper to temporarily rely on water from other cities while letting the ground water recharge when the current ground water stock is significantly low. Water may be pumped to the aquifer to expedite the ground water recharge. When the current ground water stock is sufficiently large, ground water alone can satisfy the city's water needs regardless of current rainfall.

The second essay looks at a linkage between the prices of two substitutable resources, ethanol and oil, and tests the hypothesis that the derived demand for fuel ethanol in the US is perfectly elastic. The Johansen and Juselius multivariate cointegration methodology finds no cointegration between the ethanol price and the gasoline price over the period from January 1995 to October 2008. The GH residual-based tests for cointegration in models with regime shifts indicate that the long-run relationship between the ethanol price and the gasoline price exists with a possible structural break at the estimated break point May 2007. The MTBE phase-out, the repeal of the oxygenate requirement for reformulated gasoline, and the significantly higher gasoline price in the post-May 2007 period may have caused this break in the cointegrating relationship. Based on the obtained cointegrating relationship, the demand for fuel ethanol in the pre-May 2007 period is largely governed by government regulations such as the 1990 CAAA oxygenate requirements for oxygenated and

reformulated gasoline, while the demand for fuel ethanol in the post-May 2007 period is perfectly elastic. Given that the market operates in the perfectly elastic portion of the derived demand for fuel ethanol, the market price of ethanol is derived from the gasoline price. The linkage between the ethanol price and the gasoline price should prove useful for decision makers involved in the industry and policy makers in formulating biofuel and energy policy.

The third essay looks at water recycling in ethanol production as a means to reduce some of the ethanol pressure on the water resources. As can be seen, the plant does not recycle its waste water when outside water is relatively cheap. So recycled water may be viewed as a backstop technology for outside water in this case. Since the plant tends to overuse outside water, the government may impose a Pigouvian tax on outside water use. In case a tax cannot be imposed on outside water use, the government may subsidize water recycling. Depending on whether the output constraint holds with strict inequality in the absence of water recycling and whether the plant ends up recycling all of its waste water, a comparison is made between the amount of outside water withdrawn by the plant when there is no water recycling and the amount of outside water withdrawn by the plant when there is water recycling. It is found that water recycling always leads to a reduction in the amount of outside water withdrawn by the plant when the government taxes outside water use. This is because the tax raises the net marginal cost of using outside water. However, it is not always the case that outside water use is reduced when the government subsidizes water recycling. Given the assumed ethanol production function, the subsidy for water recycling must be sufficiently small to reduce outside water use. A subsidy for water recycling that is too large may adversely induce production expansion and lead to more outside water being used by the plant. When this occurs, water recycling may no longer be beneficial in reducing the growing pressure from the ethanol plants on the water resources.

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